Name _____

Homework #4, Math 151, Fall 2008

Instructions: Record final answers and attach pages with work. All work must be shown in order to receive credit. Exact values should be used unless stated otherwise. Simplify all results.

1. Show that the slopes of the graphs are reciprocals at the indicated points.

   a. $f(x) = x^3 \quad \left(\frac{1}{2}, \frac{1}{8}\right)$

      $f^{-1}(x) = \sqrt[3]{x} \quad \left(\frac{1}{8}, \frac{1}{2}\right)$

   b. $f(x) = \frac{4}{1+x^2}, x \geq 0 \quad (1,2)$

      $f^{-1}(x) = \sqrt{\frac{4-x}{x}} \quad (2,1)$

2. Find an equation of the tangent line at the indicated point.

   a. $f(x) = \operatorname{arcsec} x \quad \left(\sqrt{2}, \frac{\pi}{4}\right)$

   b. $x \arctan x = e^y \quad \left(1, \ln\left(\frac{\pi}{4}\right)\right)$

3. Find the derivatives of the functions.

   a. $f(x) = \arctan\sqrt{x}$

   b. $g(x) = e^x \arcsin x$

   c. $h(x) = x \arccos x - \sqrt{1-x^2}$

   d. $y = \frac{1}{2}\left[x\sqrt{4-x^2} + 4\arcsin\left(\frac{x}{2}\right)\right]$

   e. $k(t) = \arctan\left(\frac{t}{2}\right) - \frac{1}{2(t^2+4)}$

4. The radius r of a sphere is increasing at a rate of 2 inches per minute. Find the rate of change of the volume when r=6 inches and r=24 inches. Explain why the rate of change of the volume of the sphere is not constant even though $\frac{dr}{dt}$ is constant.

5. A man six feet tall walks at a rate of 5 feet per second toward a light that is 20 feet above the ground. When he is 10 feet from the base of the light, at what rate is the tip of his shadow moving? At what rate is the length of the shadow changing?

6. Use Newton's method to approximate the zeros of the function. Continue the process until two successive approximations differ by less than 0.001 (10^{-3}). Then find the zero(s) using a graphing utility and compare the results. (Excel may help on this problem.)

a. $y = x^3 - 3.9x^2 + 4.79x - 1.881$

b. $f(x) = x + \sin(x+1)$

c. The intersection of $g(x) = 2 - x^2$ and $h(x) = e^{x/2}$

7. Find any critical points of the function. Determine if extrema are found there.

a. $f(x) = x^2(x^2 - 4)$

b. $g(x) = \dfrac{4x}{x^2+1}$

c. $f(\theta) = 2\sec\theta + \tan\theta \quad [0, 2\pi)$

d. $y = x\ln(x+3)$

8. Find the absolute extrema on the closed interval.

a. $f(x) = x^2 + 2x - 4 \quad [-1, 1]$

b. $y = x\ln(x+3) \quad [0, 3]$

c. $g(x) = e^x \sin x \quad [0, \pi]$

d. $h(x) = \sqrt{x} + \cos\left(\dfrac{x}{2}\right) \quad [0, 2\pi]$