Name ____

Homework #5, Math 151, Fall 2008

Instructions: Record final answers and attach pages with work. All work must be shown in order to receive credit. Exact values should be used unless stated otherwise. Simplify all results.

1. Determine whether Rolle’s theorem can be applied to f on the closed interval [a,b]. If Rolle’s theorem can be applied, find all values of c in the open interval (a,b) such that $f'(c)=0$.

   a. $f(x)=x^2-5x+4 \quad [1,4]$

   b. $f(x)=\dfrac{x^2-1}{x} \quad [-1,1]$

   c. $f(x)=\cos\ 2x \quad \left[-\dfrac{\pi}{12},\dfrac{\pi}{6}\right]$

   d. $f(x)=2+\arcsin(x^2-1) \quad [-1,1]$

2. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a,b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a,b) such that $f'(c)=\dfrac{f(b)-f(a)}{b-a}$. Use that information to find the equation of the tangent line at $c, f(c)$.

   a. $f(x)=x\ \ x^2-x-2 \quad [-1,1]$

   b. $f(x)=x^3 \quad [0,1]$

   c. $f(x)=x\log_2 x \quad [1,2]$

   d. $f(x)=2e^{x/4}\cos\left(\dfrac{\pi x}{4}\right) \quad [0,2]$

3. Find the critical numbers of f (if any). Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. Use a graph to confirm the results.

a. $f(x) = (x+2)^2(x-1)$

b. $f(x) = (x-1)^{1/3}$

c. $f(x) = (x-1)e^x$

d. $f(x) = \dfrac{\sin x}{1+\cos^2 x}$

4. Find the points of inflection. Find the intervals where the function is concave up and concave down.

a. $f(x) = x\sqrt{x+1}$

b. $f(x) = 2\csc\left(\dfrac{3x}{2}\right) \quad 0, 2\pi$

c.

d. $f(x) = \dfrac{1}{2}\ e^x - e^{-x}$

5. Find all the relative extrema. Use the 2nd derivative test where applicable.

a. $f(x) = -\dfrac{1}{8}\ x+2\ ^2\ x-4\ ^2$

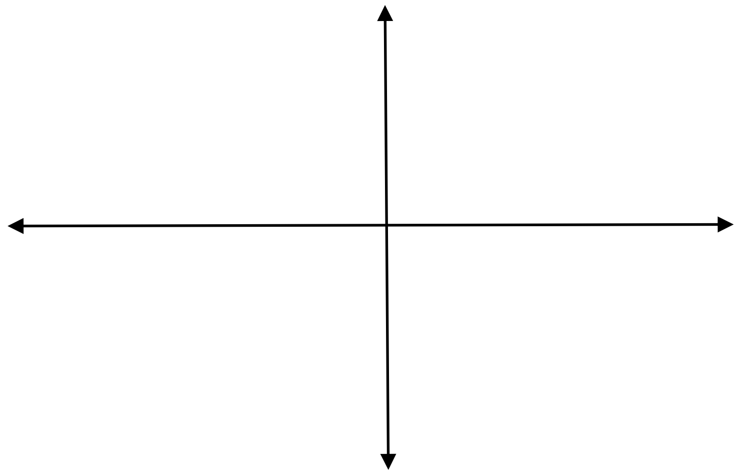
b. $g(x) = 2\sin x + \cos(2x) \quad [0, 2\pi]$

c. $h(x) = \dfrac{1}{\sqrt{2\pi}} e^{-(x-3)^2/2}$

d. $a(t) = t^2 e^{-t}$

e. $b(s) = s^2 \log_3 s$

6. Sketch the graph of a function with the following characteristics.
   i. f(1)=f(3)=0
   ii. $f'(x) < 0$ if x<2
   iii. $f'(2)$ does not exist
   iv. $f'(x) > 0$ if x>2
   v. $f''(x) < 0$ for x≠2



7. Analyze and sketch a graph of the function. Label intercepts, relative extrema, points of inflection, and asymptotes. Verify your results with a graphing utility.
   a. $f(x) = \dfrac{x+2}{x}$

   b. $g(x) = x^4 - 8x^3 + 18x^2 - 16x + 5$

   c. $h(t) = (3-t)3^t$

8. A rectangular page is to contain 30 square inches of print. The margins on each side are to be 1 ½ inches. Find the dimensions of the page such that the least amount of paper is used.

9. Fifty elk are introduced into a game preserve. It is estimated that their population will increase according to the model $p(t) = \dfrac{250}{1+4e^{-t/3}}$, where t is measured in years. At what rate is the population increasing at t=2? After how many years is the population increasing most rapidly?

10. Find the differential dy of the given functions.

a. $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

b. $y = \dfrac{\sec^2 x}{x^2 + 1}$

11. Use differentials to approximate the value of the expression. Compare to actual values from calculator.

a. $\sqrt[3]{26}$

b. 2.99^3

12. Find the indefinite integrals and check the result by differentiation.

a. $\int 4x^3 + 6x^2 - 1 \; dx$

b. $\int \theta^2 + \sec^2 \theta \; d\theta$

c. $\int \dfrac{\cos x}{1 - \cos^2 x} dx$

d. $\int y^2 \sqrt{y} dy$

e. $\int \left(\dfrac{4}{x} + 3^x \right) dx$

f. $\int dx$