Table of Integrals*

Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1} \qquad (1)$$

$$\int \frac{1}{x} dx = \ln x \qquad (2)$$

$$\int u dv = uv - \int v du \qquad (3)$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| \qquad (4)$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \qquad (5)$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1} + c, n \neq -1 \qquad (6)$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \qquad (7)$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \qquad (8)$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \qquad (9)$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2| \qquad (10)$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \qquad (11)$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2| \qquad (12)$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \qquad (13)$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, \ a \neq b \qquad (14)$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x| \qquad (15)$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \qquad (16)$$

Integrals with Roots

$$\int \sqrt{x-a}\, dx = \frac{2}{3} (x-a)^{3/2} \qquad (17)$$

$$\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a} \qquad (18)$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} \qquad (19)$$

$$\int x\sqrt{x-a}\, dx = \frac{2}{3} a(x-a)^{3/2} + \frac{2}{5} (x-a)^{5/2} \qquad (20)$$

$$\int \sqrt{ax+b}\, dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{ax+b} \qquad (21)$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a} (ax+b)^{5/2} \qquad (22)$$

$$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3} (x \mp 2a) \sqrt{x \pm a} \qquad (23)$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} \qquad (24)$$

$$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a \ln \left[\sqrt{x} + \sqrt{x+a} \right] \qquad (25)$$

$$\int x\sqrt{ax+b}\, dx = \frac{2}{15a^2} (-2b^2 + abx + 3a^2x^2) \sqrt{ax+b} \qquad (26)$$

$$\int \sqrt{x(ax+b)}\, dx = \frac{1}{4a^{3/2}} \left[(2ax+b)\sqrt{ax(ax+b)} - b^2 \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| \right] \qquad (27)$$

$$\int \sqrt{x^3(ax+b)}\, dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln \left| a\sqrt{x} + \sqrt{a(ax+b)} \right| \qquad (28)$$

$$\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2} x\sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right| \qquad (29)$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x\sqrt{a^2 - x^2} + \frac{1}{2} a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} \qquad (30)$$

$$\int x\sqrt{x^2 \pm a^2}\, dx = \frac{1}{3} \left(x^2 \pm a^2 \right)^{3/2} \qquad (31)$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left| x + \sqrt{x^2 \pm a^2} \right| \qquad (32)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} \qquad (33)$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} \qquad (34)$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} \qquad (35)$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2} x\sqrt{x^2 \pm a^2} \mp \frac{1}{2} a^2 \ln \left| x + \sqrt{x^2 \pm a^2} \right| \qquad (36)$$

$$\int \sqrt{ax^2+bx+c}\, dx = \frac{b+2ax}{4a} \sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2+bx+c)} \right| \qquad (37)$$

$$\int x\sqrt{ax^2+bx+c} = \frac{1}{48a^{5/2}} \left(2\sqrt{a}\sqrt{ax^2+bx+c} \left(-3b^2 + 2abx + 8a(c+ax^2) \right) + 3(b^3 - 4abc) \ln \left| b + 2ax + 2\sqrt{a}\sqrt{ax^2+bx+x} \right| \right) \qquad (38)$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}} dx = \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a(ax^2+bx+c)} \right| \qquad (39)$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}} dx = \frac{1}{a} \sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}} \ln \left| 2ax + b + 2\sqrt{a(ax^2+bx+c)} \right| \qquad (40)$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \qquad (41)$$

Integrals with Logarithms

$$\int \ln ax\, dx = x \ln ax - x \qquad (42)$$

$$\int \frac{\ln ax}{x} dx = \frac{1}{2} (\ln ax)^2 \qquad (43)$$

$$\int \ln(ax+b) dx = \left(x + \frac{b}{a} \right) \ln(ax+b) - x, a \neq 0 \qquad (44)$$

$$\int \ln(x^2+a^2)\, \mathrm{d}x = x \ln(x^2+a^2) + 2a \tan^{-1} \frac{x}{a} - 2x \qquad (45)$$

$$\int \ln(x^2-a^2)\, \mathrm{d}x = x \ln(x^2-a^2) + a \ln \frac{x+a}{x-a} - 2x \qquad (46)$$

$$\int \ln\left(ax^2+bx+c\right) dx = \frac{1}{a} \sqrt{4ac-b^2} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x \right) \ln\left(ax^2+bx+c\right) \qquad (47)$$

$$\int x \ln(ax+b) dx = \frac{bx}{2a} - \frac{1}{4} x^2 + \frac{1}{2} \left(x^2 - \frac{b^2}{a^2} \right) \ln(ax+b) \qquad (48)$$

$$\int x \ln\left(a^2 - b^2x^2\right) dx = -\frac{1}{2} x^2 + \frac{1}{2} \left(x^2 - \frac{a^2}{b^2} \right) \ln\left(a^2 - b^2x^2\right) \qquad (49)$$

Integrals with Exponentials

$$\int e^{ax} dx = \frac{1}{a} e^{ax} \qquad (50)$$

$$\int \sqrt{x} e^{ax} dx = \frac{1}{a} \sqrt{x} e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \mathrm{erf}\left(i\sqrt{ax}\right), \quad \text{where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \qquad (51)$$

$$\int x e^x dx = (x-1) e^x \qquad (52)$$

$$\int x e^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2} \right) e^{ax} \qquad (53)$$

$$\int x^2 e^x dx = \left(x^2 - 2x + 2 \right) e^x \qquad (54)$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax} \qquad (55)$$

$$\int x^3 e^x dx = \left(x^3 - 3x^2 + 6x - 6 \right) e^x \qquad (56)$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \qquad (57)$$

$$\int x^n e^{ax}\, \mathrm{d}x = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax], \quad \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t}\, \mathrm{d}t \qquad (58)$$

$$\int e^{ax^2}\, \mathrm{d}x = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \mathrm{erf}\left(ix\sqrt{a}\right) \qquad (59)$$

$$\int e^{-ax^2}\, \mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{a}} \mathrm{erf}\left(x\sqrt{a}\right) \qquad (60)$$

$$\int x e^{-ax^2}\, \mathrm{d}x = -\frac{1}{2a} e^{-ax^2} \qquad (61)$$

$$\int x^2 e^{-ax^2}\, \mathrm{d}x = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \mathrm{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2} \qquad (62)$$

f. $\int \tan 3x \ln(\cos 3x)\,dx$ Can it be done by parts? YES NO

If NO, what method would you use? Substitution $u=\ln(\cos 3x)$

If YES: u= dv=

g. $\int \sec^3 x\,dx$ Can it be done by parts? YES NO

If NO, what method would you use?

If YES: u= $\sec x$ dv= $\sec^2 x\,dx$

3. Integrate. $\int \frac{x^2}{\sqrt[3]{2x+1}}\,dx$ (10 points)

$u = x^2$ $dv = (2x+1)^{-1/3}dx$

$du = 2x$ $v = \frac{3}{2}\cdot\frac{1}{2}(2x+1)^{2/3}$

$\frac{3}{4}(2x+1)^{2/3}$

$\frac{3}{4}x^2(2x+1)^{2/3} - \int 2x\cdot\frac{3}{4}(2x+1)^{2/3}dx$

$u = x$ $dv = (2x+1)^{2/3}dx$

$du = dx$ $v = \frac{3}{5}\cdot\frac{1}{2}(2x+1)^{5/3}$

$\frac{3}{10}(2x+1)^{5/3}$

$\frac{3}{4}x^2(2x+1)^{2/3} - \frac{3}{2}x\cdot\frac{3}{10}(2x+1)^{5/3} + \int \frac{3}{2}\cdot\frac{3}{10}(2x+1)^{5/3}dx$

$\frac{3}{4}x^2(2x+1)^{2/3} - \frac{9}{20}x(2x+1)^{5/3} + \frac{9}{20}\cdot\frac{1}{2}\cdot\frac{3}{8}(2x+1)^{8/3} + C$

$$\boxed{\frac{3}{4}x^2(2x+1)^{2/3} - \frac{9x}{20}(2x+1)^{5/3} + \frac{27}{320}(2x+1)^{8/3} + C}$$

b. $\int \cos^2 x \sin^2 x\,dx$

$\frac{1}{4}\int (1+\cos 2x)(1-\cos 2x)\,dx$

$\frac{1}{4}\int (1-\cos^2 2x)\,dx$

$\frac{1}{4}\int dx - \frac{1}{4}\int \frac{1}{2}(1+\cos 4x)\,dx$

$\frac{1}{4}x - \frac{1}{8}x - \frac{1}{8}\cdot\frac{1}{4}\sin 4x + C$

$\boxed{\frac{1}{8}x - \frac{1}{32}\sin 4x + C}$

6. Integrate. $\int \frac{x^5}{\sqrt{8-x^2}}\,dx$ (10 points)

$x = \sqrt{8}\sin\theta$

$dx = \sqrt{8}\cos\theta\,d\theta$

$x^5 = 64\sqrt{8}\sin^5\theta$

$\sqrt{8-x^2} = \sqrt{8}\cos\theta$

$\int \frac{64\sqrt{8}\sin^5\theta\,\cancel{\sqrt{8}\cos\theta}\,d\theta}{\cancel{\sqrt{8}\cos\theta}}$

$= \int 64\sqrt{8}\sin^5\theta\,d\theta$

$u = \cos\theta$

$-du = \sin\theta\,d\theta$

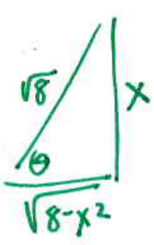
$\int 64\sqrt{8}\sin\theta(1-\cos^2\theta)^2\,d\theta$

$-64\sqrt{8}\int (1-u^2)^2\,du$

$-64\sqrt{8}\int 1-2u^2+u^4\,du = -64\sqrt{8}\left[u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right] + C$

$= -64\sqrt{8}\left[\cos\theta - \frac{2}{3}\cos^3\theta + \frac{1}{5}\cos^5\theta\right] + C$

$\frac{x}{\sqrt{8}} = \sin\theta$



$-64\sqrt{8}\left[\frac{\sqrt{8-x^2}}{\sqrt{8}} - \frac{2}{3}\left(\frac{\sqrt{8-x^2}}{\sqrt{8}}\right)^3 + \frac{1}{5}\left(\frac{\sqrt{8-x^2}}{\sqrt{8}}\right)^5\right] + C$

$\boxed{-64\sqrt{8-x^2} + \frac{16}{3}\left(\sqrt{8-x^2}\right)^3 + -\frac{1}{5}\left(\sqrt{8-x^2}\right)^5 + C}$

9. Integrate. (10 points each)

a. $\int \frac{\sec^2 x}{\tan x(\tan x+1)}dx$

$u = \tan x$

$du = \sec^2 x\, dx$

$$\int \frac{du}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

$A(u+1) + Bu = 1$

$u=0$ $\quad A = 1$

$u=-1$ $\quad -B=1 \quad B=-1$

$$\int \frac{1}{u} + \frac{-1}{u+1}\,du = \ln|u| - \ln|u+1| + C =$$

$$\boxed{\ln|\tan x| - \ln|\tan x + 1| + C}$$

b. $\int \frac{x^2+11x}{x^2+5x+6}dx$

$$\begin{array}{r} 1 \\ x^2+5x+6 \overline{\smash{)}\, x^2+11x} \\ -\ x^2+5x+6 \\ \hline 6x-6 \end{array}$$

$$\int 1 + \frac{6x-6}{(x+2)(x+3)}\,dx$$

$$\int 1 + \frac{A}{x+2} + \frac{B}{x+3}\,dx$$

$A(x+3) + B(x+2) = 6x-6$

$x=-3$

$B(-1) = -24$

$B=24$

$A(\ \ 1) = -18$

$x=-2$

$A=-18$

$$\int 1 + \frac{-18}{x+2} + \frac{24}{x+3}\,dx$$

$$= \boxed{x + 24\ln|x+3| - 18\ln|x+2| + C}$$

11. Evaluate the improper integral. Determine if it converges or diverges. If it converges, state the value. (8 points each)

a. $\int_2^{\infty} \frac{1}{\sqrt{x-1}}\,dx$

$$\lim_{b\to\infty} \int_2^b (x-1)^{-1/2}\,dx$$

$$\lim_{b\to\infty} 2(x-1)^{1/2}\Big|_2^b = \lim_{b\to\infty} 2(b-1)^{1/2} - 2(2-1)^{1/2} = \infty$$

$\boxed{\text{diverges}}$

b. $\int_{-\infty}^{\infty} xe^{x^2}\,dx$

$$\lim_{a\to-\infty} \int_{+a}^{0} xe^{x^2}\,dx + \lim_{b\to\infty} \int_0^{\infty} xe^{x^2}\,dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\lim_{a\to\infty} \frac{1}{2}e^{x^2}\Big|_a^0 + \lim_{b\to\infty} \frac{1}{2}e^{x^2}\Big|_0^b$$

$$\lim_{x\to-\infty} \frac{1}{2}e^0 - \frac{1}{2}e^{a^2} + \lim_{b\to\infty} \frac{1}{2}e^{b^2} - \frac{1}{2}e^0$$

diverges

$\boxed{\text{diverges}}$