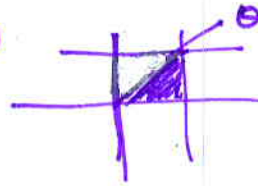
Math 2153, Quiz #12, Fall 2012 Name KEY

**Instructions:** Show all work. Use exact answers unless otherwise specifically instructed.

1. Convert the triple integral $\int_0^1 \int_y^1 \int_0^{\sqrt{1-y^2}} \sqrt{x^2+y^2}\,dz\,\cancel{dy\,dx}$ dxdy in rectangular coordinates to ones in cylindrical and spherical coordinates. Do not integrate.

$$\int_{\pi/4}^{\pi/2}\int_0^{\sec\theta}\int_0^{\sqrt{1-r^2\sin^2\theta}} r\cdot r\,dz\,dr\,d\theta$$

$z=\sqrt{1-y^2} = z=\sqrt{1-r^2\sin^2\theta}$



$\theta=\pi/4 \Rightarrow y=x$

$x=1 \Rightarrow$

$1=r\cos\theta$

$r=\sec\theta$

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$\sqrt{x^2+y^2}=\sqrt{\rho^2\sin^2\varphi}=\rho\sin\varphi$

$$\int_{\pi/4}^{\pi/2}\int_0^{\pi/2}\int_0^{\frac{1}{\sqrt{\cos^2\varphi+\sin^2\varphi\sin^2\theta}}} \rho\sin\varphi\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$x=1 \Rightarrow \rho\sin\varphi\cos\theta=1$

$\rho=\csc\varphi\sec\theta$

$\rho\cos\varphi=\sqrt{1-\rho^2\sin^2\varphi\sin^2\theta}$

$\rho^2\cos^2\varphi = 1-\rho^2\sin^2\varphi\sin^2\theta$

$\rho^2\cos^2\varphi+\rho^2\sin^2\varphi\sin^2\theta=1$

$\rho=\frac{1}{\sqrt{\cos^2\varphi+\sin^2\varphi\sin^2\theta}}$

2. Calculate the Jacobian for the change of coordinates from rectangular to cylindrical.

$x=r\cos\theta$

$y=r\sin\theta$

$z=w$

$$\frac{\partial(x,y,z)}{\partial(r,\theta,w)}=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial\theta} & \frac{\partial x}{\partial w}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\theta} & \frac{\partial y}{\partial w}\\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial\theta} & \frac{\partial z}{\partial w}\end{vmatrix}=$$

$$\begin{vmatrix}\cos\theta & -r\sin\theta & 0\\ \sin\theta & r\cos\theta & 0\\ 0 & 0 & 1\end{vmatrix} = \cos\theta(r\cos\theta-0)-(-r\sin\theta)(\sin\theta - 0)+0(0-0)$$

$$r\cos^2\theta + r\sin^2\theta = r(\cos^2\theta+\sin^2\theta)$$

$$=\boxed{r}$$