Math 2153, Quiz #14, Fall 2012 Name KEY

Instructions: Show all work. Use exact values unless specifically asked to round.

1. Evaluate the line integral $\int_C (x + 4\sqrt{y})ds$ on the line segment from (4,8) to (3,3).

$3-8=-5$
$3-4=-1$

$\vec{r}(t) = (4-t)\hat{i} + (8-5t)\hat{j} \quad 0 \le t \le 1$

$\vec{r}'(t) = -1\hat{i} - 5\hat{j}$

$\|\vec{r}'(t)\| = \sqrt{1^2+5^2} = \sqrt{26}$

$\int_0^1 (4-t+4\sqrt{8-5t})\sqrt{26}\,dt = 4t - \frac{1}{2}t^2 + -\frac{4}{5}\cdot\frac{2}{3}(8-5t)^{3/2}\Big|_0^1 =$

$u = 8-5t$
$du = -5\,dt$
$-\frac{1}{5}du = dt$

$4 - \frac{1}{2} - \frac{8}{15}(3)^{3/2} - 0 + 0 + \frac{8}{15}(8)^{3/2}$

$\boxed{\frac{7}{2} + \frac{8}{15}(8^{3/2} - 3^{3/2})}$

2. Evaluate the line integral $\int_C (3y - x)dx + y^2dy$ on the elliptic path $x = 4\sin(t), y = 3\cos(t)$ from (0,3) to (4,0).

$0 \le t \le \pi/2$

$r(t) = 4\sin t\,\hat{i} + 3\cos t\,\hat{j}$

$r'(t) = \underbrace{4\cos t}_{dx}\,\hat{i} - \underbrace{3\sin t}_{dy}\,\hat{j}$

$\int_0^{\pi/2} (3\cdot 3\cos t - 4\sin t)\cdot 4\cos t\,dt + 9\cos^2 t(-3\sin t)\,dt$

$\int_0^{\pi/2} 36\cos^2 t - 16\sin t\cos t - 27\cos^2 t\sin t\,dt$

$\int_0^{\pi/2} 18 + 18\cos 2t - 16\sin t\cos t - 27\cos^2 t\sin t\,dt =$

$18t + 9\sin 2t - 8\sin^2 t + 9\cos^3 t\Big|_0^{\pi/2} = 9\pi + 0 - 8 + 0 - 0 - 0 + 0 - 9$

$= \boxed{9\pi - 17}$

3. Use the Fundamental Theorem of Line Integrals to evaluate $\int_C -\sin(x)\,dx + zdy + ydz$ on the smooth curve from (0,0,0) to $\left(\frac{\pi}{2}, 3, 4\right)$.

$\int -\sin x\,dx = \cos x + C(y,z) \quad \int z\,dy = zy + C(x,z) \quad \int y\,dz = zy + C(x,y)$

$f(x,y,z) = \cos x + yz$

$\underbrace{\cos(\pi/2)}_{0} + 3\cdot 4 - \cos(0) - 0\cdot 0 = 12 - 1 = \boxed{11}$