

Instructions: Show all work. Use exact values unless specifically asked to round.

1. Evaluate the line integral $\int_C (x + 4\sqrt{y}) ds$ on the line segment from (4,8) to (3,3).

$$\vec{r}(t) = (4-t)\hat{i} + (8-5t)\hat{j} \quad 0 \leq t \leq 1 \quad \begin{array}{l} 3-8=-5 \\ 3-4=-1 \end{array}$$

$$\vec{r}'(t) = -1\hat{i} - 5\hat{j}$$

$$\|\vec{r}'(t)\| = \sqrt{1+25} = \sqrt{26}$$

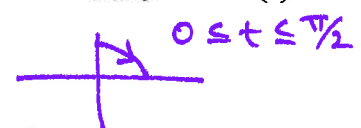
$$\int_0^1 (4-t + 4\sqrt{8-5t}) \sqrt{26} dt = 4t - \frac{1}{2}t^2 + \frac{-4}{5} \cdot \frac{2}{3} (8-5t)^{3/2} \Big|_0^1 =$$

$$4 - \frac{1}{2} - \frac{8}{15} (7)^{3/2} - 0 + 0 + \frac{8}{15} (8)^{3/2}$$

$$\boxed{\frac{7}{2} + \frac{8}{15} (8^{3/2} - 7^{3/2})}$$

2. Evaluate the line integral $\int_C (3y-x)dx + y^2 dy$ on the elliptic path $x = 4\sin(t)$, $y = 3\cos(t)$ from (0,3) to (4,0).

$$\vec{r}(t) = 4\sin t \hat{i} + 3\cos t \hat{j}$$

$$\vec{r}'(t) = 4\cos t \hat{i} - 3\sin t \hat{j}$$


$$\int_0^{\pi/2} (3 \cdot 3\cos t - 4\sin t) \cdot 4\cos t dt + \int_0^{\pi/2} 9\cos^2 t (-3\sin t) dt$$

$$\int_0^{\pi/2} 36\cos^2 t - 16\sin t \cos t - 27\cos^2 t + 3\sin t dt$$

$$\int_0^{\pi/2} 18 + 18\cos 2t - 16\sin t \cos t - 27\cos^2 t + 3\sin t dt =$$

$$18t + 9\sin 2t - 8\sin^2 t + 9\cos^3 t \Big|_0^{\pi/2} = 9\pi + 0 - 8 + 0 - 0 - 0 + 0 - 9$$

$$= \boxed{9\pi - 17}$$

3. Use the Fundamental Theorem of Line Integrals to evaluate $\int_C -\sin(x) dx + z dy + y dz$ on the smooth curve from (0,0,0) to $(\frac{\pi}{2}, 3, 4)$.

$$\int -\sin x dx = \cos x + C(x,y,z) \quad \int z dy = zy + C(x,z) \quad \int y dz = zy + C(x,y)$$

$$f(x,y,z) = \cos x + yz$$

$$\cos\left(\frac{\pi}{2}\right) + 3 \cdot 4 - \cos(0) - 0 \cdot 0 = 12 - 1 = \boxed{11}$$