Math 2153, Quiz #7, Fall 2012 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the equation of the tangent plane for the function $xy^2 + 3x - z^2 = 4$ at the point P(2,1,-2). Then find the equation of the normal line at the same point in vector-valued function form.

$F(x,y,z) = xy^2+3x-z^2-4$

$\nabla F = \langle y^2+3, 2xy, -2z \rangle = \langle 1+3, 4, -4 \rangle = \langle 4, 4, -4 \rangle$

plane: $4(x-2)+4(y-1)-4(z+2)=0$

line: $\dfrac{x-2}{4} = \dfrac{y-1}{4} = \dfrac{z+2}{-4}$ symmetric

or $\vec{r}(t) = (4t+2)\hat{i} + (4t+1)\hat{j} + (2-4t)\hat{k}$ vector function

2. Find the critical points for the function $f(x,y) = y^3 - 3yx^2 - 3y^2 - 3x^2 + 1$. Characterize each critical point as maximum, minimum, saddle point or cannot be determined.

$f_y = 3y^2 - 3x^2 - 6y = 0$

$f_x = -6xy - 6x = 0$

$-6x(y+1)=0$

$x=0$ or $y=-1$

$3y^2-6y=0$

$3y(y-2)=0$

$y=0 \quad y=2$

$(0,0) \quad (0,2)$

$3(-1)^2-3x^2-6(-1)=0$

$-3x^2+3+6=0$

$9=3x^2$

$3=x^2$

$x=\pm\sqrt{3}$

$(\sqrt{3},-1), (-\sqrt{3},-1)$

$f_{xx} = -6y-6$

$f_{yy} = 6y-6$

$f_{xy} = -6x$

$D(0,0) = (-6)(-6) - 0^2 = 36 > 0$

$(0,0)$ is a maximum

$D(0,2) = (-12-6)(12-6) - 0^2 < 0$
$(-18)(6)$

$(0,2)$ is a saddle point

$D(\sqrt{3},-1) = (6-6)(-6-6) - (-6\sqrt{3})^2 < 0$
$(0)(-12)$

$(\sqrt{3},-1)$ saddle point

$D(-\sqrt{3},-1) = (0)(-12) - (6\sqrt{3})^2 < 0$

$(-\sqrt{3},-1)$ saddle point