Math B1131, Exam #1, Fall 2012 Name KEY

Instructions: Show all work to receive full credit. Answers should be given as exact values unless specifically asked to round. You should do any obvious simplifications, but there is no need to be excessive.

You may use the following formulas on the exam:

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

1. Find the value of the sum $\sum_{k=1}^{25}(5k^2 + 3k)$. (10 points)

$$= 5\sum_{k=1}^{25} k^2 + 3\sum_{k=1}^{25} k =$$

$$5\left[\frac{25(26)(51)}{6}\right] + 3\left[\frac{25(26)}{2}\right] = 28,600$$

2. Suppose that the profit (in dollars) of q units of a product is given by $P(q) = 397q - 2.3q^3 - 400$. Use differentials to approximate the change in profit from selling q=90 items to selling q=91 items. Compare this value to the true change. (11 points)

$P'(q) = 397 - 6.9q^2$

$dq = 1$

$dP = (397 - 6.9(90^2))(1) = \boxed{\$-55,493 \Leftarrow dy}$ estimate

$P(91) = -1,697,486.3$

$P(90) = -1,641,370$

$P(91) - P(90) = \boxed{\$-56116.3 \Leftarrow \Delta y}$ true

3. Use the limit process to find the value of the area on the given interval and bounded by the function and the x-axis. $f(x) = 2x + 1, [0,2]$. Check your solution by using the Fundamental Theorem of Calculus. (25 points)

① $\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

② $x_i = a + i\Delta x = 0 + \frac{2i}{n} = \frac{2i}{n}$

③ $f(x_i) = 2\left(\frac{2i}{n}\right) + 1 = \frac{4i}{n} + 1$

④ $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \left(\frac{4i}{n} + 1\right)\left(\frac{2}{n}\right) = \sum_{i=1}^{n} \frac{8i}{n^2} + \frac{2}{n}$

⑤ $\frac{8}{n^2}\sum_{i=1}^{n} i + \sum_{i=1}^{n} \frac{2}{n} = \frac{\cancel{8}^{4}}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) + \frac{2}{\cancel{n}}\cancel{n} =$

$\frac{4\cancel{n}}{\cancel{n}} + \frac{4}{n} + 2 = 4 + \frac{4}{n} + 2 = 6 + \frac{4}{n}$

⑥ $\lim_{n\to\infty} 6 + \frac{4}{n} = \boxed{6}$ ✓

FTC. $\int_0^2 2x + 1\,dx = x^2 + x \Big|_0^2 = 4 + 2 - 0 = \boxed{6}$ ✓

4. Integrate. (12 points each)

a. $\int 5r^7 + 2r^4 + 1dr$

$$\boxed{\frac{5}{8}r^8 + \frac{2}{5}r^5 + r + C}$$

b. $\int (3y^2 + 6y)(y^3 + 3y^2 + 4)^{\frac{2}{3}}dy$

$u = y^3 + 3y^2 + 4$

$du = (3y^2 + 6y)\,dy$

$$\int u^{2/3}\,du = \frac{3}{5}u^{5/3} + C = \boxed{\frac{3}{5}(y^3+3y^2+4)^{5/3} + C}$$

c. $\int \frac{3^{\ln(x)}}{x}dx$

$u = \ln x$

$du = \frac{1}{x}$

$$\int 3^u\,du = \frac{3^u}{\ln 3} + C = \boxed{\frac{3^{\ln x}}{\ln 3} + C}$$

d. $\int u^e + e^u du$

$$\frac{u^{e+1}}{e+1} + e^u + C$$

5. Find y and the values of all the constants given $y' = e^x - 2x$ for the initial conditions $y(0) = 5$. (12 points)

$$\int e^x - 2x\,dx = e^x - x^2 + C$$

$$5 = e^0 - 0^2 + C$$

$$5 = 1 + C$$

$$C = 4$$

$$\boxed{y = e^x - x^2 + 4}$$

6. Integrate. (12 points each)

a. $\int_9^{36} \sqrt{x} - 2dx$

$$x^{1/2} - 2$$

$$\frac{2}{3}x^{3/2} - 2x \Big|_9^{36} = \frac{2}{3}(36)^{3/2} - 2(36) - \frac{2}{3}(9)^{3/2} + 2(9) =$$

$$\frac{2}{3}\cdot 216 - 72 - \frac{2}{3}27 + 18 =$$

$$144 - 72 - \cancel{18} + \cancel{18} = \boxed{72}$$

b. $\int_0^1 \frac{1}{1+e^{-x}}dx$ [Hint: multiply by $\frac{e^x}{e^x}$.]

$$\int_0^1 \frac{e^x}{e^x+1}dx \qquad u = e^x + 1 \qquad du = e^x dx \qquad \int \frac{du}{u} = \ln|u|$$

$$\Downarrow$$

$$\ln|e^x+1| \Big|_0^1 = \ln|e^1+1| - \ln|e^0+1| =$$

$$\boxed{\ln|e+1| - \ln 2}$$

7. Use Simpson's rule to approximate the value of the integral $\int_1^2 \ln(x)\,dx$, with n=4. (20 points)

$\Delta x = \frac{2-1}{4} = \frac{1}{4} = \frac{1}{4} = h$ 1, 1.25, 1.5, 1.75, 2

$f(x) \approx \frac{1/4}{3}\left[f(1) + 4f(1.25) + 2f(1.5) + 4f(1.75) + f(2)\right] =$

$\frac{1}{12}\left[\cancel{\ln 1}^{\,0} + 4\ln 1.25 + 2\ln(1.5) + 4\ln 1.75 + \ln(2)\right]$

$= .386259\ldots$

(compare to calculator value w/ fnInt: $.38629\ldots$)

8. Find the area between the curves $y = 2x, y = x(x-3)^2$. Sketch the region. (15 points)

$2x = x(x-3)^2$

$2x - x(x-3)^2 = 0$

$x\left[2 - (x-3)^2\right] = 0$ $\quad x = 0$

$2 - x^2 + 6x - 9 = 0$

$-x^2 + 6x - 7 = 0$

$x^2 - 6x + 7 = 0$

$\sqrt{2} = \sqrt{(x-3)^2}$

$\pm\sqrt{2} = x - 3$

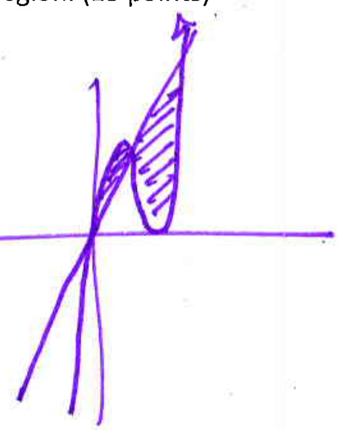
$x = 3 \pm \sqrt{2}$

$\int_0^{3-\sqrt{2}} x(x-3)^2 - 2x\,dx + \int_{3-\sqrt{2}}^{3+\sqrt{2}} 2x - x(x-3)^2\,dx$

$= \int_0^{3-\sqrt{2}} x^3 - 6x^2 + 7x\,dx + \int_{3-\sqrt{2}}^{3+\sqrt{2}} -x^3 + 6x^2 - 7x\,dx =$

$\left.\frac{x^4}{4} - 2x^3 + \frac{7}{2}x^2\right|_0^{3-\sqrt{2}} + \left(\left.-\frac{x^4}{4} + 2x^3 - \frac{7}{2}x^2\right|_{3-\sqrt{2}}^{3+\sqrt{2}}\right.$

$\downarrow$



$$\frac{(3-\sqrt{2})^4}{4} - 2(3-\sqrt{2})^3 + \frac{7}{2}(3-\sqrt{2})^2 - 0 \ +$$

$$-\frac{(3+\sqrt{2})^4}{4} + 2(3+\sqrt{2})^3 - \frac{7}{2}(3+\sqrt{2})^2 - \left(-\frac{(3-\sqrt{2})^4}{4} + 2(3+\sqrt{2})^3 - \frac{7}{2}(3+\sqrt{2}\right.$$

$$\approx \ 2.40685\ldots + 11.3137\ldots$$

$$= \ 13.72056\ldots$$