

**Instructions:** show all work. Use exact answers unless explicitly asked to do otherwise.

1. Find  $y$  subject to the given initial conditions.

a.  $y' = -x^2 + 2x$ ,  $y(2) = 1$

$$y = \int -x^2 + 2x \, dx$$

$$C = -\frac{1}{3}$$

$$y = -\frac{x^3}{3} + x^2 + C$$

$$1 = -\frac{8}{3} + 4 + C$$

$$\boxed{y = -\frac{x^3}{3} + x^2 + (-\frac{1}{3})}$$

b.  $y'' = x + 1$ ,  $y'(0) = 3$ ,  $y(1) = 16$

$$y' = \int x + 1 \, dx$$

$$y' = \frac{x^2}{2} + x + C$$

$$C = 3$$

$$y' = \frac{x^2}{2} + x + 3$$

$$y = \int \frac{x^2}{2} + x + 3 \, dx$$

$$y = \frac{x^3}{6} + \frac{x^2}{2} + 3x + C$$

$$16 = \frac{1}{6} + \frac{1}{2} + 3 + C$$

$$C = \frac{37}{3}$$

$$\boxed{y = \frac{x^3}{6} + \frac{x^2}{2} + 3x + \frac{37}{3}}$$

2. Find the anti-derivatives.

c.  $\int 3e^{3x} \, dx$

$$\frac{3e^{3x}}{3} + C = \boxed{e^{3x} + C}$$

OR explicitly:

$$u = 3x$$

$$du = 3dx \rightarrow \frac{1}{3}du = dx$$

$$\int \frac{1}{3} \cdot 3e^u \, du = \int e^u \, du = e^u + C$$

$$= e^{3x} + C$$

d.  $\int x(x^2 + 1)^3 \, dx$

$$= \int \frac{1}{2} u^3 \, du$$

$$u = x^2 + 1$$

$$du = 2x \, dx$$

$$\frac{1}{2} du = x \, dx$$

$$\frac{1}{2} \cdot \frac{u^4}{4} + C$$

$$\boxed{\frac{1}{8} (x^2 + 1)^4 + C}$$