Math 1149 Trig Identities Key

i. $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = \sec^2\theta - \tan^2\theta = 1$

Since $1 + \tan^2\theta = \sec^2\theta$ (Pythagorean identity).

ii. $3\sin^2 K + 4\cos^2 K = 3 + \cos^2 K$

$\Downarrow$

$3(1-\cos^2 K) + 4\cos^2 K = 3 - 3\cos^2 K + 4\cos^2 K =$

$3 + \cos^2 K$ ✓

iii. $1 - \dfrac{\sin^2\delta}{1-\cos\delta} = 1 - \dfrac{1-\cos^2\delta}{1-\cos\delta} = 1 - \dfrac{\cancel{(1-\cos\delta)}(1+\cos\delta)}{\cancel{1-\cos\delta}}$

$= 1 - (1+\cos\delta) = \cancel{1} - \cancel{1} - \cos\delta = -\cos\delta$ ✓

iv. $\dfrac{\cos\eta + 1}{\cos\eta - 1} = \dfrac{\frac{1}{\sec\eta} + 1}{\frac{1}{\sec\eta} - 1} \cdot \dfrac{\sec\eta}{\sec\eta} = \dfrac{1 + \sec\eta}{1 - \sec\eta}$ ✓

v. $\dfrac{\sin\rho}{\sin\rho - \cos\rho} = \dfrac{\cancel{\sin\rho}}{\cancel{\sin\rho}\left(1 - \frac{\cos\rho}{\sin\rho}\right)} = \dfrac{1}{1 - \cot\rho}$ ✓

vi. $\dfrac{\cot\lambda}{1-\tan\lambda} + \dfrac{\tan\lambda}{1-\cot\lambda} = \left[\dfrac{\frac{1}{\tan\lambda}}{1-\tan\lambda} + \dfrac{\tan\lambda}{1-\frac{1}{\tan\lambda}}\right] \dfrac{\tan\lambda}{\tan\lambda} =$

$\dfrac{1}{\tan\lambda(1-\tan\lambda)} + \dfrac{\tan^2\lambda}{\tan\lambda - 1} = \dfrac{1}{\tan\lambda(1-\tan\lambda)} - \dfrac{\tan^2\lambda}{1-\tan\lambda} \cdot \dfrac{\tan\lambda}{\tan\lambda}$

$= \dfrac{1 - \tan^3\lambda}{\tan\lambda(1-\tan\lambda)} = \dfrac{\cancel{(1-\tan\lambda)}(1+\tan\lambda+\tan^2\lambda)}{\tan\lambda\,\cancel{(1-\tan\lambda)}} = \dfrac{1+\tan\lambda+\tan^2\lambda}{\tan\lambda}$

factor difference of cubes

$= \dfrac{1}{\tan\lambda} + \dfrac{\tan\lambda}{\tan\lambda} + \dfrac{\tan^2\lambda}{\tan\lambda} = \cot\lambda + 1 + \tan\lambda$ ✓

vii. $\csc\varphi - \cot\varphi = \frac{1}{\sin\varphi} - \frac{\cos\varphi}{\sin\varphi} = \frac{1-\cos\varphi}{\sin\varphi} \cdot \frac{1+\cos\varphi}{1+\cos\varphi}$ 2

$= \frac{1-\cos^2\varphi}{\sin\varphi(1+\cos\varphi)} = \frac{\sin^{\cancel{2}}\varphi}{\cancel{\sin\varphi}(1+\cos\varphi)} = \frac{\sin\varphi}{1+\cos\varphi}$ ✓

viii. $\frac{\tan\omega - \cot\omega}{\tan\omega + \cot\omega} + 1 = \frac{\tan\omega \cancel{- \cot\omega} + \tan\omega + \cancel{\cot\omega}}{\tan\omega + \cot\omega} =$

$\frac{2\tan\omega}{\tan\omega + \cot\omega} = \frac{2\tan\omega}{\tan\omega + \frac{1}{\tan\omega}} \cdot \frac{\tan\omega}{\tan\omega} = \frac{2\tan^2\omega}{\tan^2\omega + 1} =$

$\frac{2\tan^2\omega}{\sec^2\omega} = 2\tan^2\omega \cdot \cos^2\omega = 2\frac{\sin^2\omega}{\cancel{\cos^2\omega}} \cdot \cancel{\cos^2\omega} =$

$2\sin^2\omega$. ✓

ix. $\frac{1+\sin\psi}{1-\sin\psi} - \frac{1-\sin\psi}{1+\sin\psi} = \frac{(1+\sin\psi)^2 - (1-\sin\psi)^2}{1-\sin^2\psi} =$

$\frac{1+2\sin\psi + \sin^2\psi - (1-2\sin\psi + \sin^2\psi)}{1-\sin^2\psi} =$

$\frac{\cancel{1}+2\sin\psi + \cancel{\sin^2\psi} \cancel{-1} + 2\sin\psi - \cancel{\sin^2\psi}}{\cos^2\psi} = \frac{4\sin\psi}{\cos^2\psi} =$

$4\frac{\sin\psi}{\cos\psi} \cdot \frac{1}{\cos\psi} = 4\tan\psi\sec\psi$. ✓

x. $\frac{\sin^3\tau + \cos^3\tau}{\sin\tau + \cos\tau} = \frac{\cancel{(\sin\tau + \cos\tau)}(\sin^2\tau - \sin\tau\cos\tau + \cos^2\tau)}{\cancel{\sin\tau + \cos\tau}}$

$= \sin^2\tau + \cos^2\tau - \sin\tau\cos\tau = 1 - \sin\tau\cos\tau$. ✓

$$\text{xi. } \frac{(2\cos^2\varepsilon - 1)^2}{\cos^4\varepsilon - \sin^4\varepsilon} = \frac{\cos^2 2\varepsilon}{\underbrace{(\cos^2\varepsilon + \sin^2\varepsilon)}_{=1}(\cos^2\varepsilon - \sin^2\varepsilon)} =$$

$$\frac{\cos^2 2\varepsilon}{\cos^2\varepsilon - \sin^2\varepsilon} = \frac{\cos^{\cancel{2}} 2\varepsilon}{\cancel{\cos 2\varepsilon}} = \cos 2\varepsilon = 1 - 2\sin^2\varepsilon .$$

$$\text{xii. } \frac{\tan\alpha + \tan\beta}{\cot\alpha + \cot\beta} = \frac{\tan\alpha + \tan\beta}{\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}} = \frac{\tan\alpha + \tan\beta}{\frac{\tan\beta + \tan\alpha}{\tan\alpha\tan\beta}} =$$

$$\frac{\cancel{\tan\alpha + \tan\beta}}{1} \cdot \frac{\tan\alpha\tan\beta}{\cancel{\tan\beta + \tan\alpha}} = \tan\alpha\tan\beta$$

$$\text{xiii. } \sin\left(\tfrac{3\pi}{2} + \theta\right) = \sin\left(2\pi - \tfrac{\pi}{2} + \theta\right) = \sin\left(2\pi - \left(\tfrac{\pi}{2} - \theta\right)\right)$$
$$= \sin\left(-\left(\tfrac{\pi}{2} - \theta\right)\right) = -\sin\left(\tfrac{\pi}{2} - \theta\right) = -\cos(\theta)$$

$$\text{XIV. } \sec(\mu + v) = \frac{\csc\mu\csc v}{\cot\mu\cot v - 1} = \frac{\frac{1}{\sin\mu}\cdot\frac{1}{\sin v}}{\frac{\cos\mu}{\sin\mu}\cdot\frac{\cos v}{\sin v} - 1} \cdot \frac{\sin\mu\sin v}{\sin\mu\sin v}$$

$$= \frac{1}{\cos\mu\cos v - \sin\mu\sin v} = \frac{1}{\cos(\mu + v)} = \frac{1}{\sec(\mu + v)}$$

$$\text{XV. } \frac{\sin(3\sigma)}{\sin\sigma} - \frac{\cos(3\sigma)}{\cos\sigma} = \frac{\sin(\sigma + 2\sigma)}{\sin\sigma} - \frac{\cos(\sigma + 2\sigma)}{\cos\sigma} =$$

$$\frac{\sin\sigma\cos 2\sigma + \cos\sigma\sin 2\sigma}{\sin\sigma} - \frac{\cos\sigma\cos 2\sigma - \sin\sigma\sin 2\sigma}{\cos\sigma} =$$

$$\cancel{\cos 2\sigma} + \frac{\cos\sigma \cdot 2\cancel{\sin\sigma}\cos\sigma}{\cancel{\sin\sigma}} - \cancel{\cos 2\sigma} + \frac{\sin\sigma \cdot 2\sin\sigma\cancel{\cos\sigma}}{\cancel{\cos\sigma}}$$

$$= 2\cos^2\sigma + 2\sin^2\sigma = 2(\cos^2\sigma + \sin^2\sigma) = 2$$

xvi. $\cos\varsigma = \dfrac{1-\tan^2(\varsigma/2)}{1+\tan^2(\varsigma/2)} = \dfrac{1-\tan^2(\varsigma/2)}{\sec^2(\varsigma/2)} =$

$\left(1-\tan^2(\varsigma/2)\right)\cdot\cos^2(\varsigma/2) = \left(1-\dfrac{\sin^2(\varsigma/2)}{\cos^2(\varsigma/2)}\right)\cos^2(\varsigma/2)$

$\cos^2(\varsigma/2) - \sin^2(\varsigma/2) = \cos(2\cdot\varsigma/2) = \cos\varsigma$

xvii. $\sin^2\varsigma\cos^2\varsigma = \frac{1}{8}[1-\cos(4\sigma)] \Rightarrow$

$\frac{1}{8}[1-\cos(4\varsigma)] = \frac{1}{8}[1-(1-2\sin^2 2\varsigma)] = \frac{1}{8}[\cancel{1}-\cancel{1}+2\sin^2 2\varsigma]$

$= \frac{1}{8}[2\sin^2 2\varsigma] = \frac{1}{4}[\sin 2\varsigma]^2 = \frac{1}{4}[2\sin\varsigma\cos\varsigma]^2$

$= \cancel{\frac{1}{4}}[\cancel{4}\sin^2\varsigma\cos^2\varsigma] = \sin^2\varsigma\cos^2\varsigma.$