STAT 1450, Exam #2, Fall 2013 Name KEY

Instructions: Show all work. You may use the attached formula sheet or your calculator, however, if you use your calculator, state the functions you used and the arguments entered, or I will not be able to give you partial credit. Round all probabilities to 2 decimal places (up to 4 significant figures) or use exact values. Round other values as appropriate.

1. Explain the difference between an empirical probability and a theoretical probability. (6 points)

An empirical probability is calculated from collecting many samples and figuring the proportion of results that turn up.
A theoretical probability is calculated from properties of the object, like coin flips and generally requires all probabilities be equal for simple events.

2. If the probability of event A occurring is 47%, answer the following:
   a. What is the probability that event A will **not** occur? (3 points)

   53%

   b. What are the odds **against** A? (4 points)

   $\frac{53}{47}$ or 53:47 (This does not reduce)

3. Suppose that you flip a coin 4 times, and that the coin is fair.
   a. How many ways can you come up with to get exactly two heads and two tails? (5 points)

   TTHH HHTT THTH HTHT HTTH THHT

   6 ways.

   b. What is the probability of getting exactly two heads? (4 points)

   there are 16 simple coin flips for 4 coins so $\frac{6}{16} = \frac{3}{8}$

   Or you can calculate as binomial w/ $n=4$, $p=.5$ and $x=2$

4. Define an *independent event*. Give an example of two events that are independent. (6 points)

2 events are independent if the probability of one does not influence the probability of the 2nd event.

Ex. coin flips and drawing a queen from a well-shuffled deck of cards. One does not make the other more or less likely.

5. Define a *mutually exclusive or disjoint event*. Give an example of two events that are mutually exclusive. (6 points)

mutually exclusive events are events that cannot occur simultaneously.

Ex. Having green eyes and simultaneously having blue eyes.

Answer questions 6-8 based on the table below.

	Blue Eyes	Green Eyes	Brown Eyes	Total
No credit cards	12	15	22	49
1 credit card	17	23	45	85
2 or more credit cards	91	82	135	308
Total	120	120	202	442

6. Find the probability that a randomly selected person in this sample has blue eyes. (5 points)

$\frac{120}{442} = 27.15\%$

7. Find the probability that a randomly selected person in this sample has both green eyes and 2 or more credit cards. (7 points)

$\frac{82}{442} = 18.55\%$

8. Find the probability that a randomly selected person in this sample has either brown eyes or 1 credit card. (8 points)

P(Brown) + P(1 card) − P(Brown + 1 card)

$= \frac{202}{442} + \frac{85}{442} - \frac{45}{442} = \frac{242}{442} = 54.75\%$

9. Give an example of a random variable that is discrete, and one that is continuous. Be sure to label which is which. (6 points)

discrete must be whole #'s like coin flip's, people, etc.

continuous must be able to take on any value like decimals, so: IQ, height, time, etc.

10. Determine if the following table presents a probability distribution. Why or why not? (6 points)

x	P(x)
0	0.1
1	0.2
2	0.25
3	0.28
4	0.14
5	0.03

adds to 1.00

yes. no probability is negative, and total adds to 1.

11. The following table does represent a discrete probability distribution. Find the mean and the standard deviation. (12 points)

x	P(x)
0	0.16
1	0.27
2	0.38
3	0.19

$0*.16 + 1*.27 + 2*.38 + 3*.19 = 1.6 = \text{mean } \bar{x}$

$S = .9695$

12. Suppose that the probability of having a girl is 51%.
   a. Calculate the probability of having no more than three girls if a family has 14 children. (7 points)

binomialcdf(14, .51, 3) = .0241... 2.41%

   b. What is the probability of having exactly 7 girls in a family of 14 children? (5 points)

binomialpdf(14, .51, 7) = .20888...
20.89%

13. Suppose that 24% of M&M's candies are blue. Suppose that you buy a packet of 20 candies that has no blue candies in it. Find the mean and the standard deviation of blue candies in a 20-candy packet. Is getting no blue candies unusual? (10 points)

$\mu = 20 * .24 = 4.8$

$\sigma = \sqrt{20 * .24 * .76} = 1.9099$

$2\sigma = 3.8$ usual values (1, 8.6)

Yes, getting no blues is unusual.

14. Between 1 p.m. and 4 p.m. the number of cars arriving at the pick-up window of a certain McDonald's averages car for every ten minutes. Use this information to compute the following:
   a. How many cars should the pick-up window see every hour, on average? (3 points)

6

   b. What is the probability that the pick-up window will have 10 cars or more ~~on~~ in any given ~~day~~ hour? (6 points)

1 − poissoncdf(6, 9) = .0839...

8.39%

15. Why is it not possible to calculate the probability of getting an IQ score of exactly 120? (6 points)

because IQ is continuous so the probability of that exact score is zero. You have to specify a tolerance range around it like $119.5 \le x \le 120.5$.

16. Find the area (probability) under the standard normal curve associated with the following z-scores. (4 points each)
   a. Greater than z=0.4

   normalcdf(.4, E99) = .34457... 34.46%

   b. Less than z=1.32

   normal cdf(-E99, 1.32) = .90658...

   90.66%

   c. Between z= -0.7 and z=2.1

   normalcdf(-.7, 2.1) = .74017... 74.02%

17. Find the x-values associated with the following probabilities, given the mean is μ=26 and the standard deviation σ=3.6. (5 points each)
   a. This x-value is greater than 75% of the data.

   invnorm(.75, 26, 3.6) = 28.43

   b. This x-value is less than 1% of the data.

   invnorm(.01, 26, 3.6) = 17.63

   c. We are 95% confident that the next randomly selected data point will be between what two x-values? (equally spaced on either side of the mean)

   upper = invnorm(.975, 26, 3.6)
   lower = invnorm(.025, 26, 3.6)

   (18.94, 33.06)

   d. If a certain x-value is greater than 35% of the population, calculate this value **by hand** using the z-score z= -.3853.

$$z = \frac{x-\mu}{\sigma} \Rightarrow z\sigma = x-\mu \Rightarrow z\sigma + \mu = x$$

$$-.3853 * (3.6) + 26 = 24.61$$

18. The state of Vermont, according to one study, has a mean IQ of 117, based on a sample of 1000 randomly selected people from Vermont. The standard deviation for the IQ test given is 15. (7 points each)

a. Find the confidence interval for the mean IQ in Vermont. 95%

$\frac{15}{\sqrt{1000}} = .4743...$ $1.96 * .4743 = .9296$

$(116.07, 117.93) \approx (116, 118)$

b. What does this suggest about the likelihood that people in Vermont actually score higher on IQ tests than in other parts of the country? Is 117 unusual if the mean IQ really is 100?

this suggests it is very likely Vermonters do really score above average on IQ tests. if the mean was really 100, 117 would be 35.8 standard deviations from the mean w/ this sample size.

19. Define an unbiased estimator. Give an example of a biased estimator. (6 points)

an unbiased estimator is when you take samples of data, the statistic used to estimate the population parameter clusters around the true value.

the standard deviation is a biased estimator (though the variance is not)

Stat 1450 Formulas

Unit 2

Class Width = $\dfrac{\max - \min}{\# classes}$

Class Midpoint = $\dfrac{lower + upper}{2}$

Unit 3

Range = maximum – minimum

Coefficient of Variation = $\dfrac{S}{\bar{x}} \cdot 100\%$

Chebyshev's Theorem:

At least $1 - \dfrac{1}{k^2}$ of the data lie within k standard deviations of the mean.

IQR = $Q_3 - Q_1$

z-score = $\dfrac{x - mean}{stdev} = \dfrac{x - \mu}{\sigma}$

Unit 4

P(A or B) = P(A) + P(B) – P(A and B)

P(A and B) = P(A) * P(B) if A and B are independent events

P(A and B) = P(A) * P(B|A) if A and B are dependent events

P(complement of E) = 1-P(E)

P(at least 1) = 1-P(none)

P(A|B) = $\dfrac{P(AandB)}{P(B)}$

Unit 5

Binomial: $\mu = np$ $\sigma = \sqrt{npq}$ $q = 1 - p$

Poisson: $\sigma = \sqrt{\mu}$

Unit 6

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{(\bar{x} - \mu)}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

Unit 7

Proportions:

$CI = \hat{p} \pm z \cdot \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$ or $\hat{p} - E \le p \le \hat{p} + E$

$$n = \frac{z^2 \hat{p}\hat{q}}{E^2} = \hat{p}\hat{q}\left(\frac{z}{E}\right)^2$$

Means:

$CI = \bar{x} \pm z \cdot \dfrac{\sigma}{\sqrt{n}}$ or $\bar{X} - E \le \mu \le \bar{X} + E$

$$n = \left(\frac{z * \sigma}{E}\right)^2$$

Unit 8

Hypothesis Testing:

Proportions $z = \dfrac{(\hat{p} - p)}{\sqrt{pq/n}}$

Means $z = \dfrac{(\bar{x} - \mu)}{\left(\frac{\sigma}{\sqrt{n}}\right)}$