Name KEY

Math 2568, Final Exam – Part I, Fall 2013

Instructions: On this portion of the exam, you may **NOT** use a calculator. Show all work. Answers must be supported by work to receive full credit.

1. The system shown below is in vector equation form.

$$x_1\begin{bmatrix}2\\1\end{bmatrix} + x_2\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix}$$

a. Write the system as a matrix equation. (3 points)

$$\begin{bmatrix}2 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}4\\5\end{bmatrix}$$

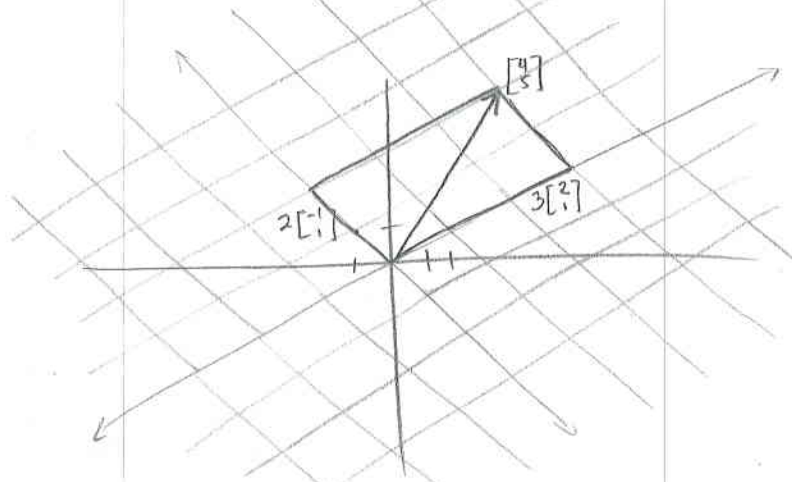
b. Solve the system by using an inverse matrix. Write the solution as a column vector. (8 points)

$\text{Det } A = 2+1 = 3$

$$A^{-1} = \frac{1}{3}\begin{bmatrix}1 & 1\\-1 & 2\end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix}1 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}4\\5\end{bmatrix} = \frac{1}{3}\begin{bmatrix}4+5\\-4+10\end{bmatrix} = \frac{1}{3}\begin{bmatrix}9\\6\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix} = \begin{bmatrix}x_1\\x_2\end{bmatrix}$$

c. Use the solution you obtained and graphically represent it on a graph as the linear combination of the vectors $\begin{bmatrix}2\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$. Be sure to show the coordinate gridlines on your graph. (6 points)



2. Find the determinant of the matrix $A = \begin{bmatrix} 5 & 3 & 0 \\ 2 & -3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$ by any means. (7 points)

$$1\begin{vmatrix} 3 & 0 \\ -3 & 4 \end{vmatrix} + 1\begin{vmatrix} 5 & 3 \\ 2 & -3 \end{vmatrix} = (12+0) + (-15-6) =$$

$$12 + (-21) = -9$$

3. Find the QR factorization of the matrix A, given that $A = \begin{bmatrix} -2 & 1 \\ 4 & 7 \\ -2 & 1 \\ -5 & -5 \end{bmatrix}$ and $Q = \begin{bmatrix} -2/7 & \sqrt{3}/3 \\ 4/7 & \sqrt{3}/3 \\ -2/7 & \sqrt{3}/3 \\ -5/7 & 0 \end{bmatrix}$.

In other words, find R. (8 points)

$$R = Q^T A$$

$$\begin{bmatrix} -2/7 & 4/7 & -2/7 & -5/7 \\ \sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 & 0 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 4 & 7 \\ -2 & 1 \\ -5 & -5 \end{bmatrix} =$$

$$\begin{bmatrix} 4/7 + 16/7 + 4/7 + 25/7 & -2/7 + 28/7 - 2/7 + 25/7 \\ -\frac{2\sqrt{3}}{3} + \frac{4\sqrt{3}}{3} - \frac{2\sqrt{3}}{3} + 0 & \sqrt{3}/3 + 7\sqrt{3}/3 + 1\sqrt{3}/3 + 0 \end{bmatrix} = \begin{bmatrix} 7 & 7 \\ 0 & 3\sqrt{3} \end{bmatrix}$$

4. For the matrix $A = \begin{bmatrix} -5 & 10 & 4 & 14 \\ -7 & 11 & 5 & 13 \\ -3 & 4 & 4 & 5 \\ 2 & -2 & -2 & -1 \end{bmatrix}$. The eigenvalues are $\lambda = 1, 2, 3$ Find the eigenvectors corresponding to each eigenvalue and determine if the matrix is diagonalizable. (20 points)

repeated root can't tell here

$\lambda_1 = 1$ $\begin{bmatrix} -6 & 10 & 4 & 14 \\ -7 & 10 & 5 & 13 \\ -3 & 4 & 3 & 5 \\ 2 & -2 & -2 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\vec{V_1} = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_2 = 2$ $\begin{bmatrix} -7 & 10 & 4 & 14 \\ -7 & 9 & 5 & 13 \\ -3 & 4 & 2 & 5 \\ 2 & -2 & -2 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\vec{V_2} = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda_3 = 3$ $\begin{bmatrix} -8 & 10 & 4 & 14 \\ -7 & 8 & 5 & 13 \\ -3 & 4 & 1 & 5 \\ 2 & -2 & -2 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -3 & -3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\vec{V_3} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \vec{V_4} = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ 2-D eigenspa ok.

diagonalizable

5. Suppose that det(C)=12. Find the determinant of the matrix after the following row operations. (5 points)

$$6R_1 + R_2 \to R_2, R_4 \leftrightarrow R_3, 3R_2 + 4R_4 \to R_4, \frac{1}{4}R_4 \to R_4$$

no change (-1) $*4$ $(1/4)$

-12

6. Determine if the set formed by polynomials of the form $p(t) = a + bt + t^2$ is a subspace of P_n. If it is, prove it. If it is not, find an example to the contrary. (6 points each)

it is not a subspace

$p(t) = a + bt + t^2$
$q(t) = c + dt + t^2$ $\Rightarrow$ $p + q = (a+c) + (b+d)t + 2t^2$

not in subset

7. Consider the orthogonal basis for $\mathbb{R}^3$ given by $\left\{\begin{bmatrix}-1\\1\\3\end{bmatrix}, \begin{bmatrix}0\\3\\-1\end{bmatrix}, \begin{bmatrix}10\\1\\3\end{bmatrix}\right\}$. Use the property of Orthogonality to find the coordinate representation of the vector $\vec{x} = \begin{bmatrix}5\\7\\-1\end{bmatrix}$ in this basis. [Hint: no matrices are required.] (15 points)

$$C_1 = \frac{-5+7-3}{1+1+9} = \frac{-1}{11}$$

$$C_2 = \frac{0+21+1}{9+1} = \frac{22}{10} = \frac{11}{5}$$

$$C_3 = \frac{50+7-3}{100+1+9} = \frac{54}{110} = \frac{27}{55}$$

$$[\vec{X}]_B = \begin{bmatrix}-1/11\\11/5\\27/55\end{bmatrix}$$

1. Given the vectors $\vec{u} = \begin{bmatrix}3\\-1\\4\\5\end{bmatrix}, \vec{v} = \begin{bmatrix}-2\\6\\3\\7\end{bmatrix}$ find the following.

a. A unit vector in the direction of $\vec{v}$. (4 points)

$$\|\vec{v}\| = \sqrt{4+36+9+49} = \sqrt{98} = 7\sqrt{2}$$

$$\hat{V} = \begin{bmatrix}-2/7\sqrt{2}\\6/7\sqrt{2}\\3/7\sqrt{2}\\1/\sqrt{2}\end{bmatrix}$$

b. Find the distance between $\vec{u}$ and $\vec{v}$. (7 points)

$$\begin{bmatrix}3-(-2)\\-1-6\\4-3\\5-7\end{bmatrix} = \begin{bmatrix}5\\-7\\1\\-2\end{bmatrix} \qquad \sqrt{25+49+1+4} = \sqrt{79}$$

8. Show that the polynomials $f(t) = 1 - 2t$, and $g(t) = 8 + 3t$ are orthogonal under the inner product $< f, g > = \int_{-2}^{2} f(t)g(t)dt$. (10 points)

$$\int_{-2}^{2}(1-2t)(8+3t)dt = \int_{-2}^{2} 8 \underbrace{-16t+3t}_{odd \to 0} - 6t^2\, dt = \underbrace{2\int_{0}^{2} 8-6t^2 d}_{even}$$

$$2\left[8t - 2t^3\right]_0^2 = 2\left[16-16\right] = 0$$

Yes, they are orthogonal

9. Determine if the following sets of vectors are linearly independent by inspection. Justify your answer in each case. (5 points each)

a. $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$ Zero vector not independent

b. $\left\{\begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \underset{V_2}{\begin{bmatrix}1\\2\\0\\3\end{bmatrix}}, \underset{V_3}{\begin{bmatrix}-2\\-4\\0\\-6\end{bmatrix}}, \begin{bmatrix}3\\5\\2\\1\end{bmatrix}\right\}$ $\vec{V_3} = -2\vec{V_2}$ dependent

c. $\left\{\begin{bmatrix}2\\3\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}\right\}$ 2 vectors, not multiples, independent

10. Consider the stochastic Markov chain matrix given by the matrix $A = \begin{bmatrix} .9 & .05 \\ .1 & .95 \end{bmatrix}$. Calculate the equilibrium vector of the system. (5 points)

$\begin{bmatrix} -.1 & .05 \\ .1 & -.05 \end{bmatrix}$ $\quad \dfrac{.1x_1}{.1} = \dfrac{.05x_2}{.1} \Rightarrow x_1 = .5x_2$ or $\begin{matrix} x_1 = \frac{1}{2}x_2 \\ x_2 = x_2 \end{matrix}$

$\vec{x} = \begin{bmatrix}1\\2\end{bmatrix}$ $\quad 1+2=3 \rightarrow \vec{q} = \begin{bmatrix}1/3\\2/3\end{bmatrix}$

11. Determine if each statement is True or False. (3 points each)

a. T F If vectors $\vec{v_1}, \vec{v_2}, \ldots \vec{v_p}$ span a subspace W and if $\vec{x}$ is orthogonal to each $\vec{v_j}$ for j=1...p, then $\vec{x}$ in $W^{\perp}$.

b. T F If $\vec{y}$ is in a subspace $W^{\perp}$, then the orthogonal projection of $\vec{y}$ onto W is $\vec{y}$ itself.

c. T F Every eigenvalue has only one corresponding eigenvector.

d. T F Both $\begin{bmatrix} 1 & * & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are matrices in echelon form.

e. T F If A is a 3x5 matrix, then the transformation $\vec{x} \mapsto A\vec{x}$ can be onto but not one-to-one. 3 pivots .

f. T F If a system of equations has a free variable then it has a unique solution.

g. T F The complex eigenvalues of a discrete dynamical system either both attract to the origin or both repel from the origin.

h. T F If two vectors are orthogonal, they are linearly independent.

i. T F $\{\vec{0}\}$ is a subspace.

j. T F The third standard basis vector $\vec{e_3}$ in P_6 is t^3. that is $\vec{e_4}$

k. T F The null space of a matrix is a subspace of the codomain of the matrix. = range

l. T F If A is diagonalizable, then A is invertible. $\lambda = 0$

m. T F A matrix is invertible if and only if 0 is an eigenvalue of A. $\lambda \neq 0$

n. T F If the columns of A are linearly independent, then the equation $A\vec{x} = \vec{b}$ has an infinite number least-squares solutions, or none at all. only one

o. T F A least-squares solution of $A\vec{x} = \vec{b}$ is the point in the column space of A closest to $\vec{b}$.

p. T F An isomorphism is a linear mapping from one n-dimensional space into another space of the same number of dimensions. also must be one-to-one & onto, but this is still true

q. T F A matrix given by $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has a unique solution if $ad - bc = 0$. $\neq 0$

Name____

Math 2568, Final Exam – Part II, Fall 2013

Instructions: On this portion of the exam, you *may* use a calculator to perform elementary matrix operations. Support your answers with work (reproduce the reduced matrices from your calculator) or other justification for full credit.

1. Find a least squares solution for the set of points $\{(1,0.7),(2.1,2.9),(2.2,4.8),(3.1,9.7),(4.4,18.3),(5.3,28.8)\}$ to satisfy the equation $y = \beta_0 + \beta_1 x + \beta_2 x^2$. Be sure to write the matrices employed, any equations, and the final regression function for y. (15 points)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2.1 & 2.1^2 \\ 1 & 2.2 & 2.2^2 \\ 1 & 3.1 & 3.1^2 \\ 1 & 4.4 & 4.4^2 \\ 1 & 5.3 & 5.3^2 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} .7 \\ 2.9 \\ 4.8 \\ 9.7 \\ 18.3 \\ 28.8 \end{bmatrix}$$

$$A^T A \vec{x} = A^T \vec{b}$$

$$\vec{x} = (A^T A)^{-1} A^T \vec{b}$$

$$\vec{x} \approx \begin{bmatrix} .28 \\ -.73 \\ 1.14 \end{bmatrix}$$

$$y = .28 - .73x + 1.14x^2$$

2. Given the vectors $\vec{b_1} = \begin{bmatrix} 3 \\ 1 \\ 8 \\ 2 \end{bmatrix}$ $and\ \vec{b_2} = \begin{bmatrix} 1 \\ -3 \\ 2 \\ -8 \end{bmatrix}$, find two more vectors orthogonal to these (and each other) to make an orthogonal basis for $\mathbb{R}^4$. (15 points)

$$\begin{bmatrix} 3 & 1 & 8 & 2 \\ 1 & -3 & 2 & -8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 13/5 & -1/5 \\ 0 & 1 & 1/5 & 13/5 \end{bmatrix} \Rightarrow \begin{bmatrix} -13 \\ -1 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 13 \\ 0 \\ 5 \end{bmatrix}$$

3. The set $H = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \right\}$ forms a basis for R^3. Use the Gram-Schmidt Process to make an orthogonal basis, and then normalize it. (15 points)

$$\bar{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \frac{0+0-1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} - \frac{1+2-4}{1+1+4}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} - \frac{1+2+2}{1+1+1}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} + \frac{+1}{6}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} - \frac{5}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} + \begin{bmatrix} 1/6 \\ 1/6 \\ 1/3 \end{bmatrix} + \begin{bmatrix} -5/3 \\ -5/3 \\ +5/3 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 1/2 \\ 0 \end{bmatrix} = \qquad \vec{V}_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

orthogonal basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

orthonomal basis $\left\{ \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}, \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} \right\}$

4. Given the basis of $W = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$, and the vector $\vec{y} = \begin{bmatrix} 5 \\ 2 \\ -1 \\ 0 \end{bmatrix}$ decompose this vector into $\vec{y} = \vec{y_{\parallel}} + \vec{y_{\perp}}$ with $\vec{y_{\parallel}} = proj_W\vec{y}$. (15 points)

$$\vec{y}_{\parallel} = \frac{5+2+0+0}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \frac{0+0-1+0}{5}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \frac{7}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \frac{-1}{5}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7/2 - 0 \\ 7/2 - 0 \\ 0 - 1/5 \\ 0 - 2/5 \end{bmatrix} = \begin{bmatrix} 7/2 \\ 7/2 \\ -1/5 \\ -2/5 \end{bmatrix}$$

$$\vec{y}_{\perp} = \begin{bmatrix} 5 \\ 2 \\ -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 7/2 \\ 7/2 \\ -1/5 \\ -2/5 \end{bmatrix} = \begin{bmatrix} 3/2 \\ -3/2 \\ -4/5 \\ 2/5 \end{bmatrix}$$

5. Assume that $A = \begin{bmatrix} 1 & 3 & 0 & 5 & 0 & 3 \\ 2 & 2 & -1 & 2 & 2 & -5 \\ 1 & -1 & 3 & -3 & 1 & 9 \\ 5 & 4 & 1 & 3 & 1 & 7 \end{bmatrix}$. Find a basis for the null space of *A*. (10 points)

$$rref \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 5/19 \\ 0 & 1 & 0 & 2 & 0 & 52/57 \\ 0 & 0 & 1 & 0 & 0 & 217/57 \\ 0 & 0 & 0 & 0 & 1 & -101/57 \end{bmatrix}$$

$$\begin{aligned} x_1 &= x_4 - 5/19\, x_6 \\ x_2 &= -2x_4 - 52/57\, x_6 \\ x_3 &= -217/57\, x_6 \\ x_4 &= x_4 \\ x_5 &= 101/57\, x_6 \\ x_6 &= x_6 \end{aligned}$$

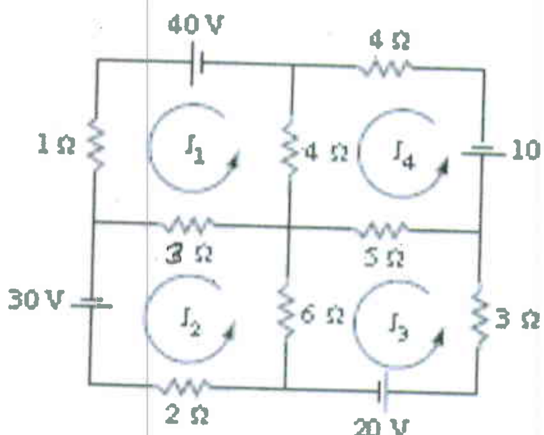
$x_4 = t$

$x_6 = 57s$

$$\vec{X} = t \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -15 \\ -52 \\ -217 \\ 0 \\ 101 \\ 57 \end{bmatrix}$$

$$Nul\, A = \left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -15 \\ -52 \\ -217 \\ 0 \\ 101 \\ 57 \end{bmatrix} \right\}$$

6. Based on the graph below, solve the system for the circuit flowing through each loop. You may round your answers to three decimal places as needed. (10 points)



$$\begin{aligned} 8I_1 - 3I_2 - 4I_4 &= 40 \\ -3I_1 + 11I_2 - 6I_3 &= 30 \\ -6I_2 + 14I_3 - 5I_4 &= 20 \\ -4I_1 - 5I_3 + 13I_4 &= -10 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 8 & -3 & 0 & -4 & 40 \\ -3 & 11 & -6 & 0 & 30 \\ 0 & -6 & 14 & -5 & 20 \\ -4 & 0 & -5 & 13 & -10 \end{array}\right]$$

$$\vec{I} = \begin{bmatrix} 11.824 \\ 10.293 \\ 7.957 \\ 5.930 \end{bmatrix} \text{ or } \begin{bmatrix} 23{,}850/2017 \\ 20{,}760/2017 \\ 16050/2017 \\ 11960/2017 \end{bmatrix}$$

7. The following are short answer questions. Always provide justification for any answers. You may use examples as part of your explanations, but if you are asked to "explain" your answer must contain **words**. (4 points each)

   a. Give an example of a 5x5 matrix with a non-trivial solution.

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

answers will vary

   b. Why must an nxn matrix have n ***distinct*** eigenvalues to guarantee that the eigenspace spans $\mathbb{R}^n$?

each eigenvalue guarantees only one eigenvector. repeated eigenvalues from the characteristic equation are not guaranteed to produce more than one eigenvector & you need n vectors to form a basis for $\mathbb{R}^n$.

   c. Give two properties of the invertible matrix theorem and explain why they must be equivalent to each other.

answers will vary:
an invertible matrix reduces to the nxn identity which has a non-zero determinant. row operations can change the determinant by magnitude or sign, but cannot make it zero. A matrix that cannot reduce to I_n has a zero on its diagonal.

   d. Give an example of a stochastic matrix that has more than one equilibrium vector.

$$\begin{bmatrix} .4 & .3 & 0 & 0 \\ .6 & .7 & 0 & 0 \\ 0 & 0 & .8 & .1 \\ 0 & 0 & .2 & .9 \end{bmatrix}$$

answers will vary.

e. Explain why the equation y=mx+b is not a linear transformation under the definitions used in this course.

it does not pass the definitions used.

Consider $2f(x) = f(2x)$? no.

Since $2f(x) = 2y = 2mx + 2b$ but

$f(2x) = m(2x) + b = 2mx + b$

These are not equal

f. Explain why the complex eigenvalues of a dynamical system cannot produce a saddle point.

the magnitude of both eigenvalues is the same size since they are complex conjugates

g. What are the advantages and disadvantages of finding determinants by row-reducing compared to the cofactor method?

fewer operations for large matrices

h. Give at least two reasons why being able to diagonalize a matrix is so important computationally.

when doing operations by hand, it can make powers of matrices much easier to compute

i. Explain the relationship between a vector $\vec{y}$ in $\mathbb{R}^n$, W a subspace of $\mathbb{R}^n$, $\vec{v}, \vec{y_{\parallel}}$ which are vectors in W, as described by the Best Approximation Theorem.

W a subspace of $\mathbb{R}^n$
$y \in \mathbb{R}^n$

$\vec{y}_{\parallel} \in W$ $\vec{v}$ is any other vector in W

The Best approximation theorem says that for a vector $\vec{y} \in \mathbb{R}^n$ then $\|\vec{y}-\vec{y}_{\parallel}\| < \|\vec{y}-\vec{v}\|$ where $\vec{y}_{\parallel}$ is the orthogonal projection of $\vec{y}$ onto W and $\vec{v}$ is any other vector in W. in other words, the point represented by $\vec{y}_{\parallel}$ is the closest point to $\vec{y}$ in W.