1. $A = \begin{bmatrix} 2 & -4 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

2. $A = \begin{bmatrix} 3 & -2 \\ 1 & 4 \\ 0 & 1 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

5. $(t^3 - 4)(a_0 + a_1 t + a_2 t^2) = a_0 t^3 + a_1 t^4 + a_2 t^5 - 4a_0 - 4a_1 t - 4a_2 t^2$

$$= -4a_0 - 4a_1 t - 4a_2 t^2 + a_0 t^3 + a_1 t^4 + a_2 t^5$$

$$T\left(\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} -4a_0 \\ -4a_1 \\ -4a_2 \\ a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$A = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. $(2t^2-t+6)(a_0 + a_1t + a_2t^2) =$

$2a_0t^2 + 2a_1t^3 + 2a_2t^4 - a_0t - a_1t^2 - a_2t^3 + 6a_0 + 6a_1t + 6a_2t^2$

$6a_0 + (6a_1 - a_0)t + (6a_2 - a_1 + 2a_0)t^2 + (2a_1 - a_2)t^3 + 2a_2t^4$

$$T\left(\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} 6a_0 \\ -a_0 + 6a_1 \\ 2a_0 - a_1 + 6a_2 \\ 2a_1 - a_2 \\ 2a_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & 0 & 0 \\ -1 & 6 & 0 \\ 2 & -1 & 6 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

6. $\frac{d}{dt}\left[a_0 + a_1t + a_2t^2 + a_3t^3\right] = a_1 + 2a_2 + 3a_3t^2$

$$T\left(\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = \begin{bmatrix} a_1 \\ 2a_2 \\ 3a_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

7. $\int a_0 + a_1 t + a_2 t^2 + a_3 t^3 \, dt =$

$$a_0 t + \tfrac{1}{2} a_1 t^2 + \tfrac{1}{3} a_2 t^3 + \tfrac{1}{4} a_3 t^4$$

$$T\left(\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ a_0 \\ \tfrac{1}{2} a_1 \\ \tfrac{1}{3} a_2 \\ \tfrac{1}{4} a_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \tfrac{1}{2} & 0 & 0 \\ 0 & 0 & \tfrac{1}{3} & 0 \\ 0 & 0 & 0 & \tfrac{1}{4} \end{bmatrix}$$

8. $\frac{d}{dx}\left[a_1 e^x + a_2 e^{-x} + a_3 e^{5x} + a_4 e^{-7x} \right] =$

$$a_1 e^x - a_2 e^{-x} + 5a_3 e^{5x} - 7a_4 e^{-7x}$$

$$T\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}\right) = \begin{bmatrix} a_1 \\ -a_2 \\ 5a_3 \\ -7a_4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

9. $\dfrac{d}{dx}\left[a_1 e^{3x}\cos 2x + a_2 e^{3x}\sin 2x\right] =$

$3a_1 e^{3x}\cos 2x - 2a_1 e^{3x}\sin 2x + 3a_2 e^{3x}\sin 2x + 2a_2 e^{3x}\cos 2x$

$(3a_1 + 2a_2) e^{3x}\cos 2x + (-2a_1 + 3a_2) e^{3x}\sin 2x$

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} 3a_1 + 2a_2 \\ -2a_1 + 3a_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}$$

10. $(9a_1 + 6a_2) e^{3x}\cos 2x + (-6a_1 - 4a_2) e^{3x}\sin 2x +$

$(-6a_1 + 9a_2) e^{3x}\sin 2x + (-4a_1 + 6a_2) e^{3x}\cos 2x$

$(5a_1 + 12a_2) e^{3x}\cos 2x + (-12a_1 + 5a_2) e^{3x}\cos 2x$

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} 5a_1 + 12a_2 \\ -12a_1 + 5a_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 12 \\ -12 & 5 \end{bmatrix}$$

11.
$$\begin{bmatrix} \cos 225^\circ & -\sin 225^\circ \\ \sin 225^\circ & \cos 225^\circ \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

12.
$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ -1 & 0 \end{bmatrix}$$

or
$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & -3 \end{bmatrix}$$

13.
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\pi/3 & -\sin 2\pi/3 \\ 0 & \sin 2\pi/3 & \cos 2\pi/3 \end{bmatrix}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & -\sqrt{3}/2 \\ 0 & \sqrt{3}/2 & -1/2 \end{bmatrix}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 4 \\ -\sqrt{3}/2 & -1 & 0 \\ -1/2 & \sqrt{3} & 0 \end{bmatrix}$$