

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. State the dimension of Nul A, Row A, Col A, and rank A.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 2 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\dim \text{Row } A = \dim \text{Col } A = \text{rank } A = 3$$

$$\dim \text{Nul } A = 5$$

2. If a 5×9 matrix has a rank of 3, state the dimensions of Nul A, Col A, Row A^T .

$$\dim \text{Nul } A = 6$$

$$\dim \text{Col } A = 3$$

$$\dim \text{Row } A^T = 3$$

3. Do the polynomials $\{1 + 3t^2, 2t - t^3, t^2 + 5t^3, 7 + t + t^3\}$ form a basis for P_3 ?

yes. $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 2 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & -1 & 5 & 1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$

4. Find the change of basis matrix for changing a C-coordinate vector into a B-coordinate vector if

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix} \right\}, C = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 1 \\ -2 \end{bmatrix} \right\}.$$

$$P_B [\vec{x}]_B = P_C [\vec{x}]_C$$

$$[\vec{x}]_B = P_B^{-1} P_C [\vec{x}]_C = P_{B \leftarrow C} [\vec{x}]_C$$

$$P_{B \leftarrow C} = \begin{bmatrix} 1 & 1 & 3 \\ 0 & -1 & 0 \\ 1 & 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 & 7 \\ 0 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 11 & 32 \\ -1 & 3 & -1 \\ 0 & -4 & -8 \end{bmatrix}$$