

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Determine if the system below has only the trivial solution or a non-trivial one. You do not have to find the solution if it is non-trivial. Show enough work to justify your answer.

$$\begin{cases} x_1 + 2x_2 - 3x_3 = 0 \\ 2x_1 + x_2 - 3x_3 = 0 \\ -x_1 + x_2 = 0 \end{cases}$$

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

this system has a nontrivial solution

2. Determine if the following sets of vectors are linearly independent or linearly dependent. You do not need to use your calculator on most of them. Justify your answer (briefly).

a.  $\left\{ \begin{bmatrix} 1 \\ 7 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} \right\}$  independent 2 vectors, not multiples

b.  $\left\{ \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix} \right\}$  dependent 3 vectors in  $\mathbb{R}^2$

c.  $\left\{ \begin{bmatrix} 1 \\ 3 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix} \right\}$  dependent only 3 pivots in matrix of vectors

3. Determine if the transformation  $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - 3x_2 + x_4 \\ 2x_1x_2 - x_3 \\ 3x_2 + 5x_3 - 2x_4 \\ x_4 \end{bmatrix}$  is linear or nonlinear using the definition of a linear transformation. (Verify all three properties if it is linear, or find a contradiction if it is not.)

This transformation is nonlinear.

Consider 2<sup>nd</sup> condition  $T(c\vec{u}) = cT(\vec{u})$

$$T(c\vec{u}) = \begin{bmatrix} 2cu_1 - 3cu_2 + u_4 \\ 2cu_1cu_2 - cu_3 \\ 3cu_2 + 5cu_3 - 2cu_4 \\ cu_4 \end{bmatrix} = \begin{bmatrix} 2cu_1 - 3cu_2 + cu_4 \\ 2c^2u_1u_2 - cu_3 \\ 3cu_2 + 5cu_3 - 2cu_4 \\ cu_4 \end{bmatrix} = c \begin{bmatrix} 2u_1 - 3u_2 + u_4 \\ 2cu_1u_2 - u_3 \\ 3u_2 + 5u_3 - 2u_4 \\ u_4 \end{bmatrix}$$

$$cT(\vec{u}) = c \begin{bmatrix} 2u_1 - 3u_2 + u_4 \\ 2u_1u_2 - u_3 \\ 3u_2 + 5u_3 - 2u_4 \\ u_4 \end{bmatrix} \text{ which is not the same.}$$