

Instructions: Show all work. Answers must be justified in some fashion to receive credit, even if you use your calculator. Given exact answers unless specifically asked to round.

1. For the transformation $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 4x_1 - 2x_2 + x_3 \\ 3x_2 + x_1 + 5x_3 \\ 0 \\ -x_1 + \frac{1}{2}x_3 \end{bmatrix}$, determine if the transformation is linear

(you do not need to use the definition to prove it here, but if it is not, show a contradiction). If it is linear, write the matrix of the transformation.

is linear

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 1 & 3 & 5 \\ 0 & 0 & 0 \\ -1 & 0 & \frac{1}{2} \end{bmatrix}$$

2. For the matrix transformation given by $A = \begin{bmatrix} 2 & 4 & 2 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \\ 3 & -1 & 0 \end{bmatrix}$, determine whether the transformation is one-to-one, onto, both, or neither.

rref \Rightarrow $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(add column of 0's to make calc. happy, then ignore it)

3 pivots, one in each column \Rightarrow
 linearly independent \Rightarrow
 one-to-one
 not onto