

Instructions: Show all work. Use exact values.

1. Explain why, in your own words, why an invertible matrix must not have a zero determinant. Be sure that you are talking about general square matrices and not specifically the 2x2 case only.

an invertible matrix must have a nonzero determinant because it must reduce to the $n \times n$ identity matrix which has a determinant of 1. Row operations can multiply the determinant by scalars but not by zero. If the determinant is zero, it won't reduce to the $n \times n$ identity.

2. Use the cofactor method to calculate the determinant of the matrix $A = \begin{bmatrix} 6 & 1 & 2 & 3 \\ 0 & -2 & 0 & -4 \\ 4 & 3 & 2 & -1 \\ 0 & 5 & 1 & 1 \end{bmatrix}$.

You may check your answer in the calculator, but you must show work to receive credit.

$$6 \begin{vmatrix} -2 & 0 & -4 \\ 3 & 2 & -1 \\ 5 & 1 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 & 3 \\ -2 & 0 & -4 \\ 5 & 1 & 1 \end{vmatrix} =$$

$$6 \left[2 \begin{vmatrix} -2 & -4 \\ 5 & 1 \end{vmatrix} - \begin{vmatrix} -2 & -4 \\ 3 & -1 \end{vmatrix} \right] + 4 \left[-2 \begin{vmatrix} -2 & -4 \\ 5 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ -2 & -4 \end{vmatrix} \right]$$

$$6 \left[2(-2+20) - (+2+12) \right] + 4 \left[-2(-2+20) - (-4+6) \right]$$

$$6 \left[2(18) - 14 \right] + 4 \left[-2(18) - (2) \right] =$$

$$6 \left[36 - 14 \right] + 4 \left[-36 - 2 \right] = 6 \left[22 \right] + 4 \left[-38 \right] =$$

$$132 - 152 = \boxed{-20}$$