Math 2568, Quiz #7, Fall 2013 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round.

1. Find the determinant of the matrix $A = \begin{bmatrix} 0 & -4 & -1 & 0 \\ 1 & 5 & -2 & 3 \\ 3 & 6 & 2 & -1 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ using the row-reducing method.

$$-1\begin{vmatrix} 1 & 5 & -2 & 3 \\ 0 & -4 & -1 & 0 \\ 3 & 6 & 2 & -1 \\ 2 & 1 & 3 & 1 \end{vmatrix} \Rightarrow \begin{matrix} -3R_1 + R_3 \to R_3 \\ -2R_1 + R_4 \to R_4 \\ -3 \ -15 \ 6 \ -9 \\ -2 \ -10 \ 4 \ -6 \end{matrix} \ -1\begin{vmatrix} 1 & 5 & -2 & 3 \\ 0 & -4 & -1 & 0 \\ 0 & -9 & 8 & -10 \\ 0 & -9 & 7 & -5 \end{vmatrix} \Rightarrow$$

$$-1(1)\begin{vmatrix} -4 & -1 & 0 \\ -9 & 8 & -10 \\ -9 & 7 & -5 \end{vmatrix} \quad \begin{matrix} -2R_3 + R_2 \to R_2 \\ 18 \quad -14 \quad 10 \end{matrix} \quad (-1)(1)\begin{vmatrix} -4 & -1 & 0 \\ 9 & -6 & 0 \\ -9 & 7 & -5 \end{vmatrix} =$$

$$(-1)(1)(-5)\begin{vmatrix} -4 & -1 \\ 9 & -6 \end{vmatrix} = (-1)(1)(-5)(24+9) = 5(33) = 165$$

2. Determine if the sets below are vector spaces.

a. $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix}, x + y \geq 0 \right\}$

not a vector space.

i. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $x + y = 0 + 0 \geq 0$ ✓

ii. $\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} x+a \\ y+b \end{bmatrix}$ $x+a+y+b \geq 0$? $(x+y) \geq 0 + (a+b) \geq 0$ yes.

iii. $k\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} kx \\ ky \end{bmatrix}$ $kx + ky \geq 0$? No $-1\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \end{bmatrix} \Rightarrow -3-2 \not\geq 0$

b. $S = \left\{ \begin{bmatrix} z \\ w \end{bmatrix}, z = a + bi, w = c + di; a, b, c, d \text{ real} \right\}$ $\begin{bmatrix} a+bi \\ c+di \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} i$

i. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ if $a = b = c = d = 0$ yes.

ii. $\begin{bmatrix} a+bi \\ c+di \end{bmatrix} + \begin{bmatrix} e+fi \\ g+hi \end{bmatrix} = \begin{bmatrix} a+e \\ c+d \end{bmatrix} + \begin{bmatrix} b+f \\ d+h \end{bmatrix} i$ $(a+e), (d+h), (b+f), (c+d)$ all real ✓

iii. $k\begin{bmatrix} a+bi \\ c+di \end{bmatrix} = \begin{bmatrix} ka + kbi \\ kc + kdi \end{bmatrix}$ yes since $(ka), (kb), kc), ((kd)$ all real. ✓

this is a vector space.