Math 2568, Quiz #8, Fall 2013 Name KEY

Instructions: Justify answers with work or explanation. Use exact values.

1. Show that the set $H = \left\{ \begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix}, \begin{matrix} 3p + r = s \\ p + q + 2r = 2s \end{matrix} \right\}$ is a subspace of R^4 or give a specific example to the contrary.

$3p + r - s = 0$

$p + q + 2r - 2s = 0$

$\Rightarrow A = \begin{bmatrix} 3 & 0 & 1 & -1 \\ 1 & 1 & 2 & -2 \end{bmatrix}$

yes. it is a vector space

$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 1/3 & -1/3 \\ 0 & 1 & 5/3 & -5/3 \end{bmatrix}$

$p = -1/3\, r + 1/3\, s$
$q = -5/3\, r + 5/3\, s$
$r = r$
$s = s$

$\begin{bmatrix} p \\ q \\ r \\ s \end{bmatrix} = c \begin{bmatrix} -1 \\ -5 \\ 3 \\ 0 \end{bmatrix} + d \begin{bmatrix} 1 \\ 5 \\ 0 \\ 3 \end{bmatrix}$

i) if $c = d = 0 \Rightarrow \vec{0}$ ✓

ii) adding $c\begin{bmatrix} -1 \\ -5 \\ 3 \\ 0 \end{bmatrix} + d\begin{bmatrix} 1 \\ 5 \\ 0 \\ 3 \end{bmatrix} + k\begin{bmatrix} -1 \\ -5 \\ 3 \\ 0 \end{bmatrix} + n\begin{bmatrix} 1 \\ 5 \\ 0 \\ 3 \end{bmatrix} = (c+k)\begin{bmatrix} -1 \\ -5 \\ 3 \\ 0 \end{bmatrix} + (d+n)\begin{bmatrix} 1 \\ 5 \\ 0 \\ 3 \end{bmatrix}$

still a linear combo of same vectors

iii) scalars $k\left[c\begin{bmatrix} -1 \\ -5 \\ 3 \\ 0 \end{bmatrix} + d\begin{bmatrix} 1 \\ 5 \\ 0 \\ 3 \end{bmatrix}\right] = (kc)\begin{bmatrix} -1 \\ -5 \\ 3 \\ 0 \end{bmatrix} + (kd)\begin{bmatrix} 1 \\ 5 \\ 0 \\ 3 \end{bmatrix}$

still linear combo ✓

2. Given $A = \begin{bmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{bmatrix}$, find each of the following: [You may want to work on the back of the page.]

a. An explicit description of Nul A.

$\text{rref} = \begin{bmatrix} 1 & 0 & 1/3 & 0 & 10/3 \\ 0 & 1 & 1/3 & 0 & -26/3 \\ 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 = -1/3\, x_3 - 10/3\, x_5$
$x_2 = -1/3\, x_3 + 26/3\, x_5$
$x_3 = x_3$
$x_4 = 4x_5$
$x_5 = x_5$

$\text{Nul } A = \text{span}\left\{ \begin{bmatrix} -1 \\ -1 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -10 \\ 26 \\ 0 \\ 12 \\ 3 \end{bmatrix} \right\}$

b. A basis for Col A.

$\text{Col } A = \text{span} \left\{ \begin{bmatrix} 5 \\ 3 \\ 8 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -5 \\ 0 \end{bmatrix} \right\}$

c. A basis for Row A.

$\text{Row } A = \text{Span} \left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 10 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \\ -26 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -4 \end{bmatrix} \right\}$