

Instructions: Justify answers with work or explanation. Use exact values.

1. Determine whether the set of vectors $\left\{ \begin{bmatrix} 1 \\ 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} -5 \\ -4 \\ 2 \\ 13 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix} \right\}$ forms a basis for \mathbb{R}^4 . If it does not, explain why not.

it does not form a basis. it does not span \mathbb{R}^4
nor is the set linearly independent.

2. Determine whether the set of vectors $\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 11 \\ 14 \\ -25 \\ 6 \end{bmatrix} \right\}$ spans \mathbb{R}^4 . If it does, find a basis for the space.

$$\text{rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 2/57 & -188/57 \\ 0 & 1 & 0 & 0 & -14/57 & 5/57 \\ 0 & 0 & 1 & 0 & 9/19 & 275/19 \\ 0 & 0 & 0 & 1 & -11/57 & -676/57 \end{bmatrix}$$

There are too many vectors
to be independent in \mathbb{R}^4 .
Confirmed by echelon form.

basis: $\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 4 \\ 1 \end{bmatrix} \right\}$

3. Determine if the polynomials $\{1 - 2t, 3 + 5t^2, -t + 2t^2\}$ form a basis for \mathcal{P}_2 . [Hint: Write them as vectors, first.]

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \right\} \text{ rref} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

yes, this forms a basis for \mathcal{P}_2 .