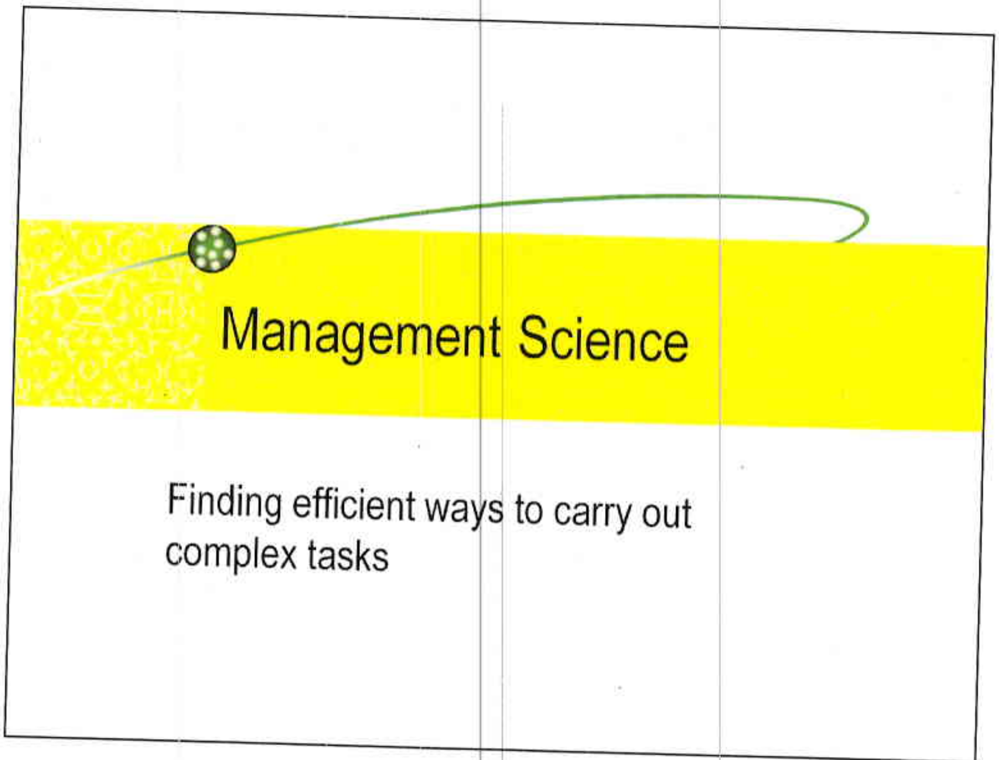

Chapter 5a.ppt

5.1 – 5.4

Management Science, dealing with tasks that involve large numbers of variables, and in which it is not obvious how to make optimal decisions.

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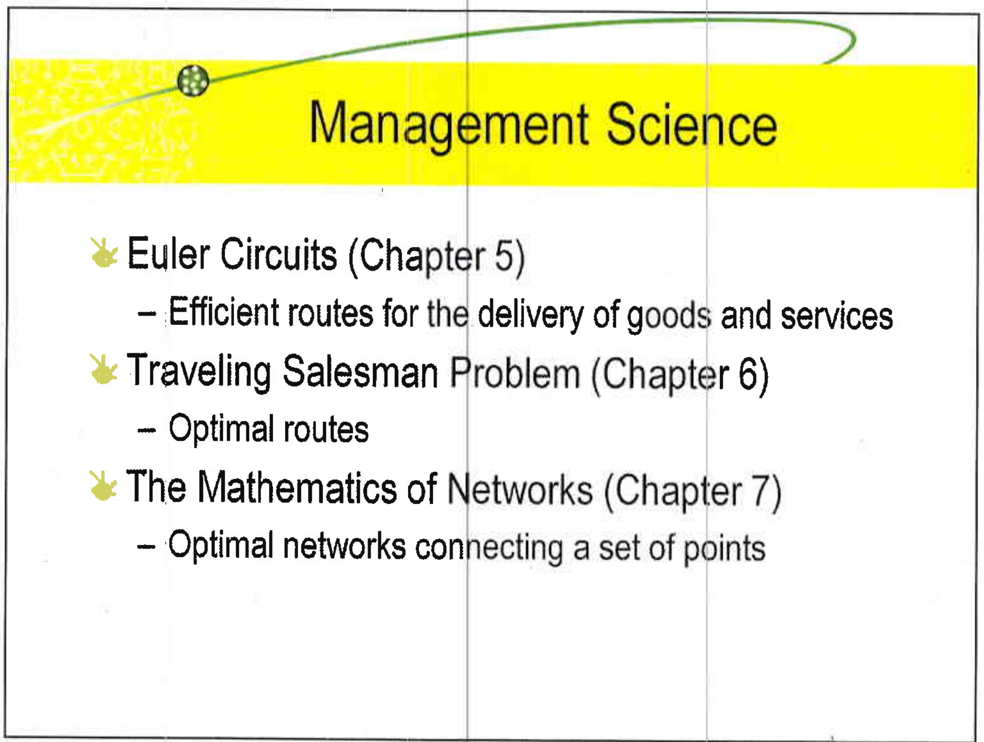
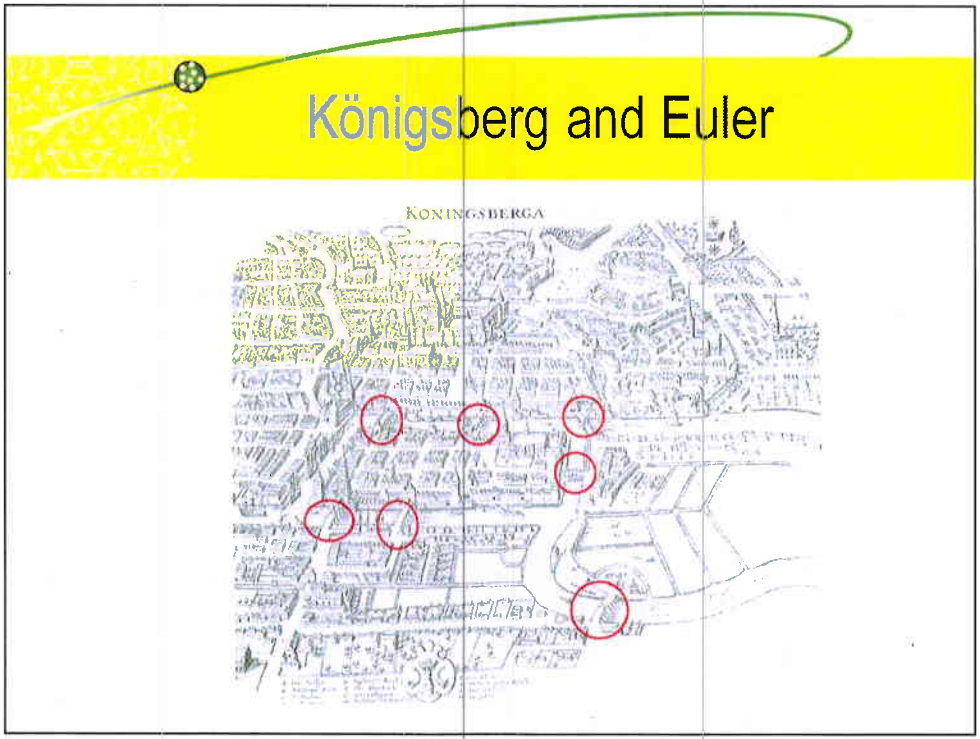


We will be dealing with *Graph Theory* – its language and theory, and some algorithms to make it work, as we look at Euler Circuits, the Traveling Salesman Problem (not Traveling Salesmen jokes) and Networks.



Königsberg (in English we would say *Kingston* – King's town). In the 1700's it was a part of Prussia (NE Germany). The city is now called Kalingrad. It is in that part of Russia which is separate from the rest, between Poland and Lituania.

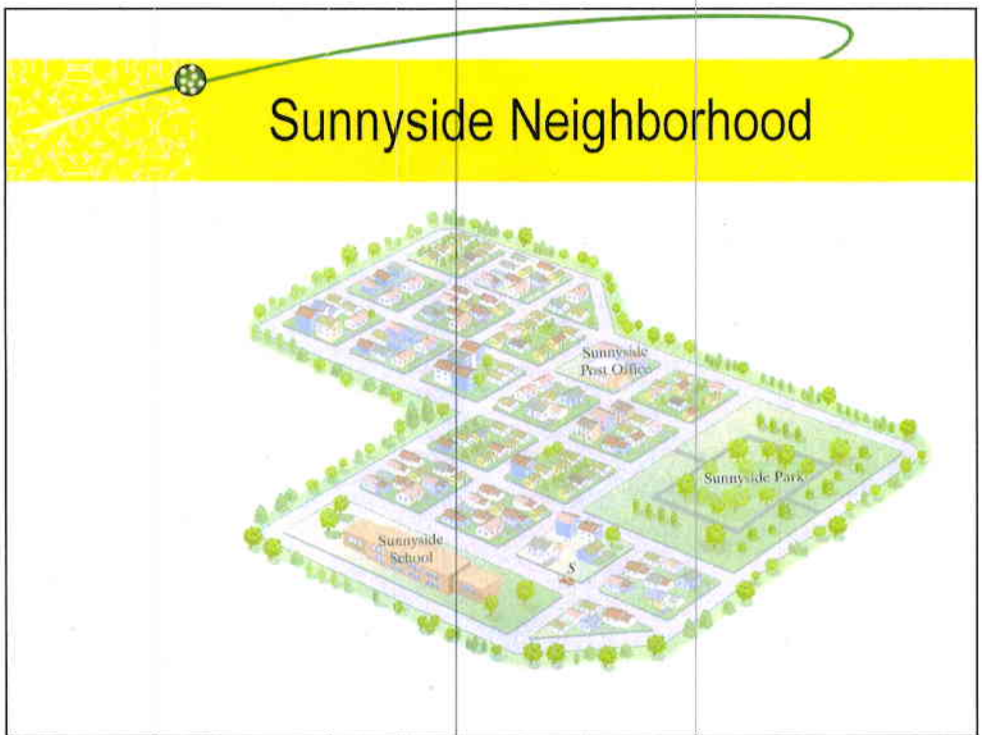
In the 1700's Königsberg was divided by a river into 4 sections and there were 7 bridges. This is a map from 1736, when the young mathematician Leonard Euler came passing through.

Question: Is it possible to tour the town, crossing each bridge exactly once? The locals had tried, repeatedly, and without success. Could he prove mathematically that it could not be done?

He could. And it gave rise to his new *geometris situs* which has developed into one of the most important and practical branches of modern matematics, called *graph theory*.

Our task: how to create the most efficient routes for delivery of goods and services (mail delivery, police patrols, garbage collection, newspaper deliveries, pizza deliveries, etc.)

Today we will look at an example of an Euler circuit problem, which we will solve in a few days; the language and concepts of graph theory; and graph modeling with which complicated real-life problems can be translated into the clean precise language of graph theory.



Here is the Sunnyside neighborhood. There are two problems here:

1. Security. The patrol wants to park at point *S* and walk every street once, returning to the car. If it can't be done, what is the optimal (shortest) route that the patrol should follow to cover every street?
2. Postal Delivery. The mail carrier must walk each street with homes once for each side of the street. That is, if there are houses on only one side of the street, like near the park, the carrier need only walk that street once. But blocks that have houses on both sides must be walked twice. What is the optimal (shortest) route to cover the entire town for mail delivery?

Here's a simpler map of Königsberg.

I will give an A for the quarter to anyone who can find a way to walk each bridge just once, and return to the original starting point.

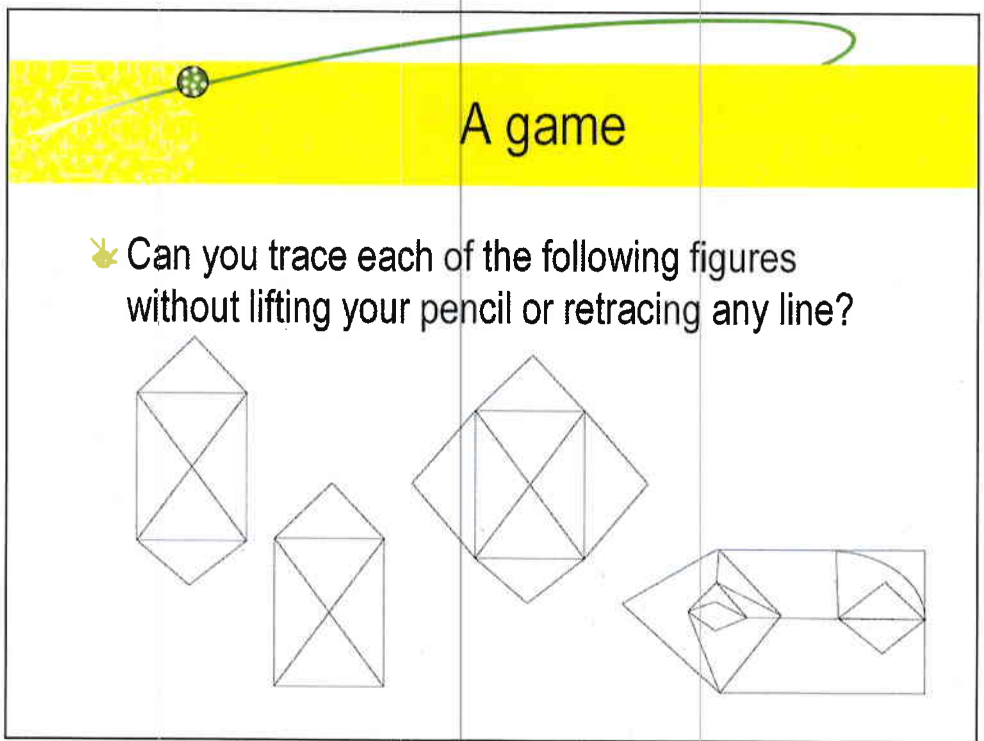
I will give an A on the next test to anyone who can cross each bridge just once without necessarily returning to the starting point.

No one has ever received either of these prizes. Let's see why…

Bridges of Madison County

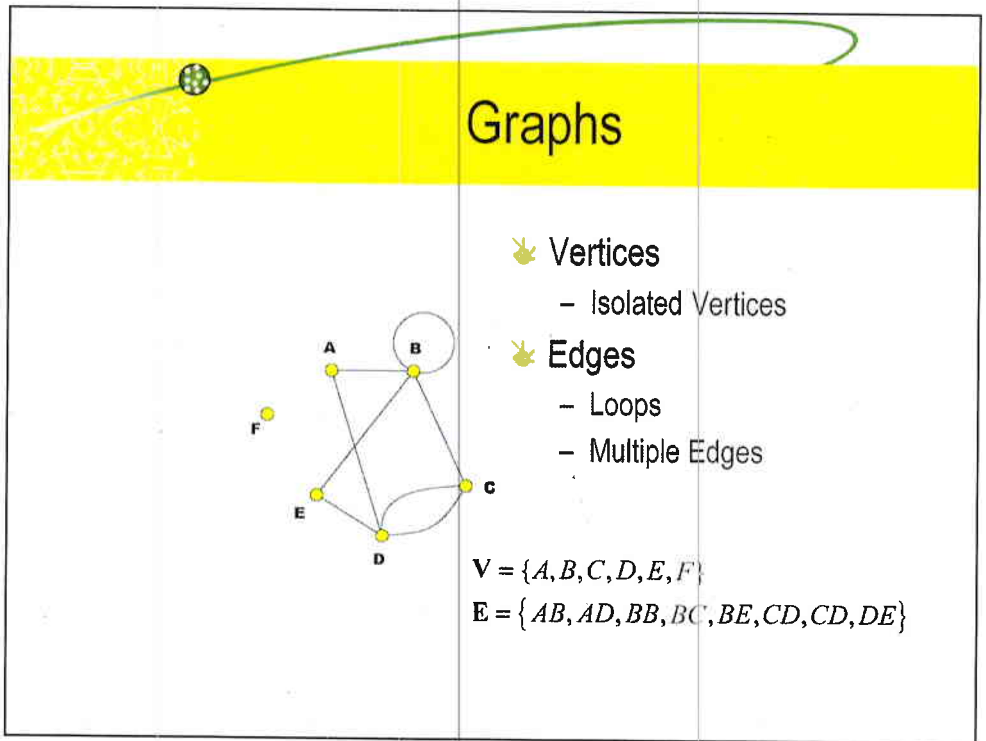
A beautiful river runs through the county. There are 4 islands (*A*, *B*, *C*, and *D*) and 11 bridges. The county charges a tax of $25 for non-residents to cross each bridge.

A photographer for a national magazine wants to photograph each of the 11 bridges. The photographer wants to cross each bridge once, and only recross it if absolutely necessary. What is the cheapest route for him to take?



See if you can trace each drawing without lifting your pencil or retracing any line.

This kind of tracing is called *unicursal* tracing. A closed unicursal tracing returns to the starting point, while an open unicursal tracing does not return to the starting point.



Vertices are the "dots" while isolated vertices are dots with no lines connecting to them. Singular of vertices is vertex.

Edges are the lines. Each edge must always connect two vertices. A loop is an edge that connects a vertex to itself. Multiple edges are two or more edges connecting the same vertices.

We use sets to describe graphs: one to indicate the vertices and one to represent the edges. Note that as far as edges are concerned $AB = BA$. Multiple edges are listed more than once, and edges like AA represent loops.

Graphs

- A graph is a *structure that defines pairwise relationships* within a set of objects. The objects are the vertices, and the pairwise relationships are the edges. X is related to Y if and only if XY is an edge.

This is the definition of a graph. ⌘

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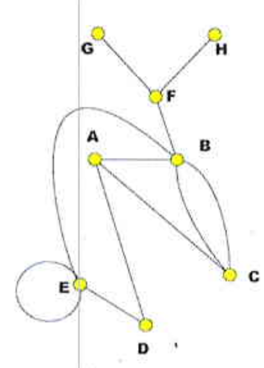
Terminology

- Adjacent Vertices
- Adjacent Edges
- Degree of a Vertex *Deg*(*V*)
- Odd and Even Vertices
- Paths
- Circuits



- Adjacent vertices have an edge joining them

- Adjacent edges share a common vertex

- Degree of a vertex = number of edge "ends" at that vertex. Note that a loop contributes 2

- Odd and Even Vertices. If deg(V) is odd, the vertex is called odd. If deg(V) is even, the vertex is called even.

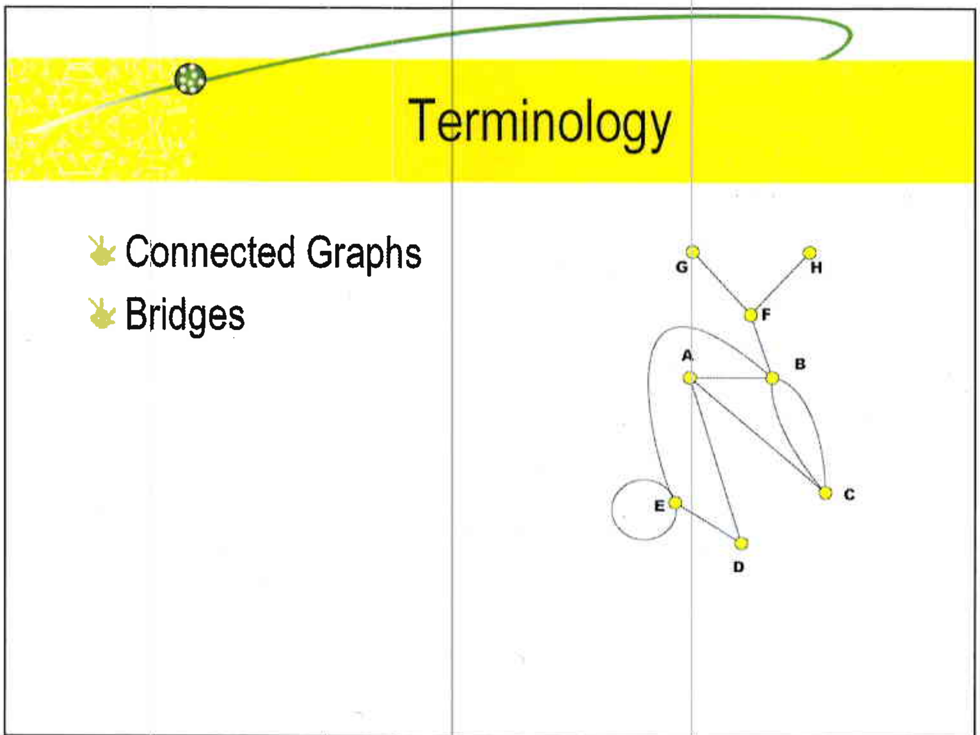
- Path: A sequence of vertices, with the property that each vertex is adjacent to the next.

  - A trip, if you will, along the edges of a graph
  - An edge can be part of a path *only once*

- The number of edges is the length of the path
- Examples: ABED / ABCADE / ABCBE / ABCEED
- But not: ACDE / ABCBAD / ABCBEEDACB (why not?)

Circuit: A path that starts and ends at the same vertex

- ABCA = BCAB = CABC
- BCB
- EE



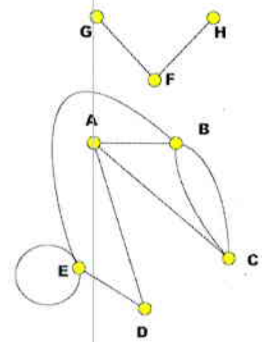
✂ Connected Graph: Given any two vertices, there is a path connecting them.

✂ Bridge: an edge that if we erase it, leaves the graph no longer connected. For example, edge BF

✂

Terminology

- Connected Graphs
- Bridges
- Disconnected Graphs



- Disconnected graph: a graph in pieces…
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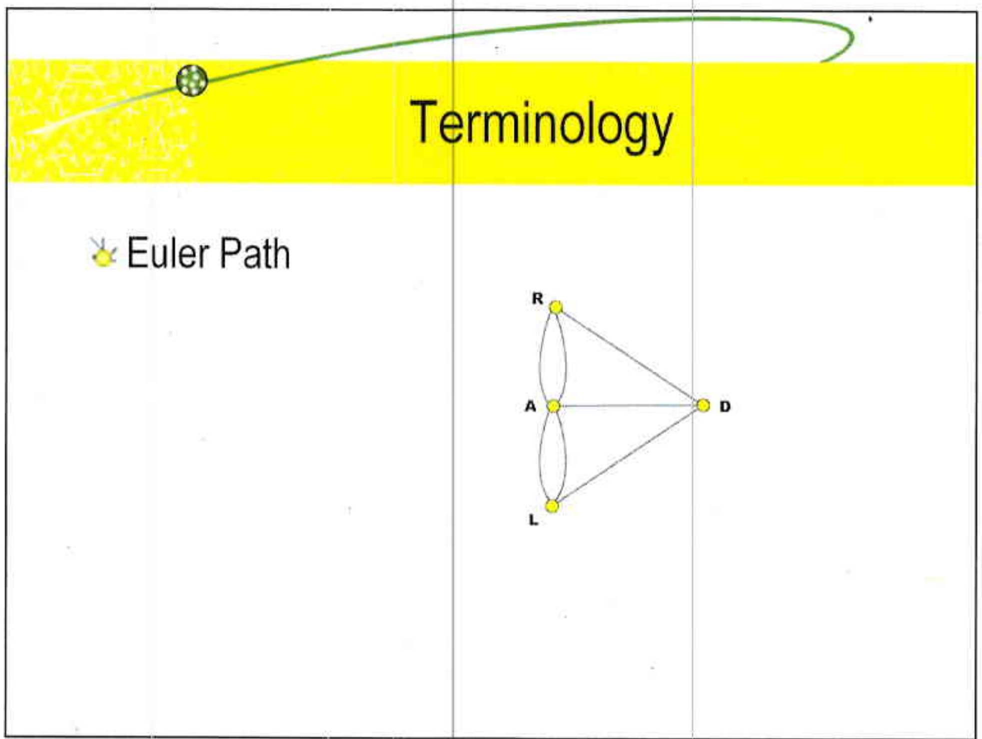
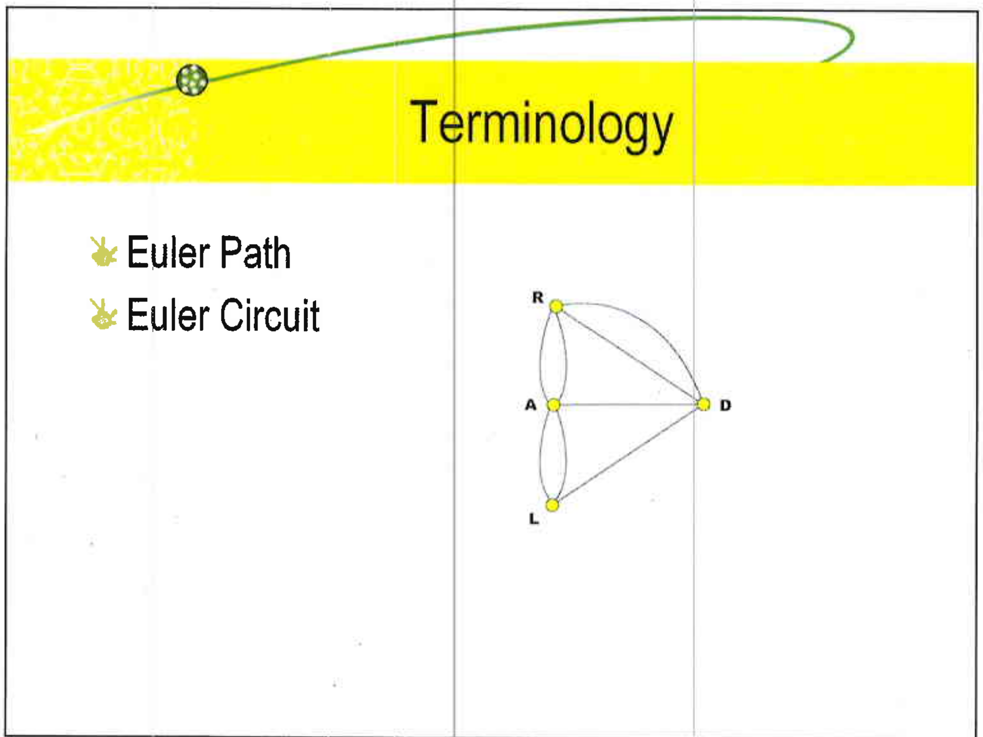


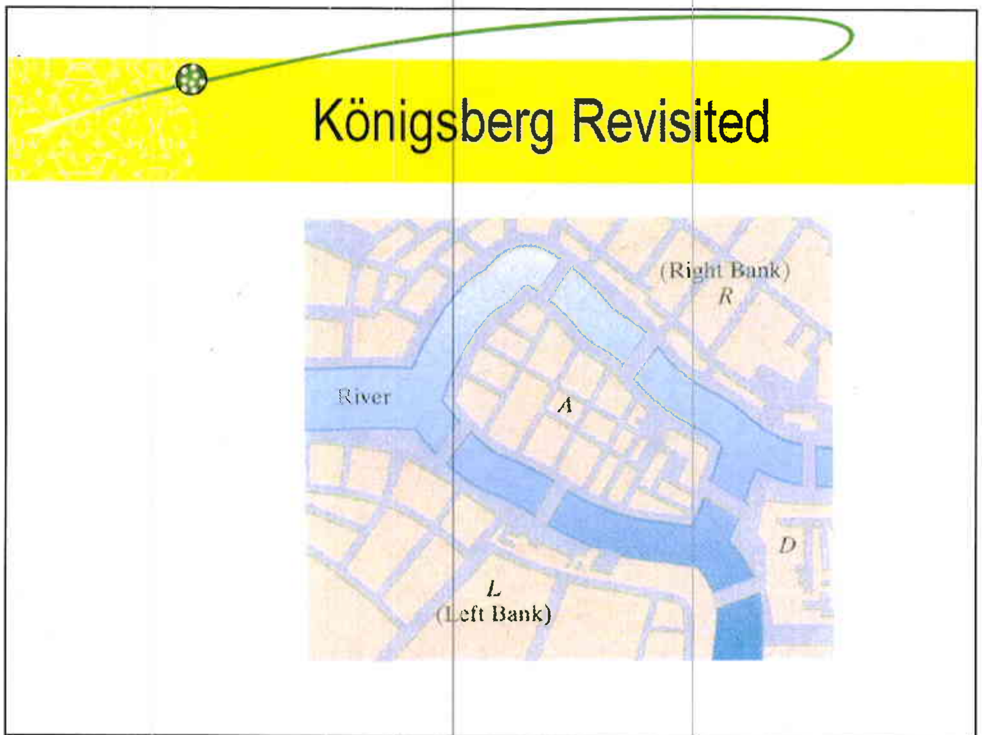
Euler Path: path that passes through every edge of the graph once and only once. Clearly, a disconnected graph cannot have an Euler Path.

This graph has no Euler Path.



This graph has several Euler paths. One is LARDARDLA. Can you find another that doesn't start at L?

An Euler Circuit is a circuit that passes through every edge of a graph exactly once, and ends at the starting point. In other words, a closed unicursal tracing of the graph. The given graph does not have an Euler circuit, but I'll put a couple on the board that have several each.



One of Euler's greatest contributions is the recognition that certain types of life problems can easily be rephrased as graph problems.

Modeling: translating real life into mathematics

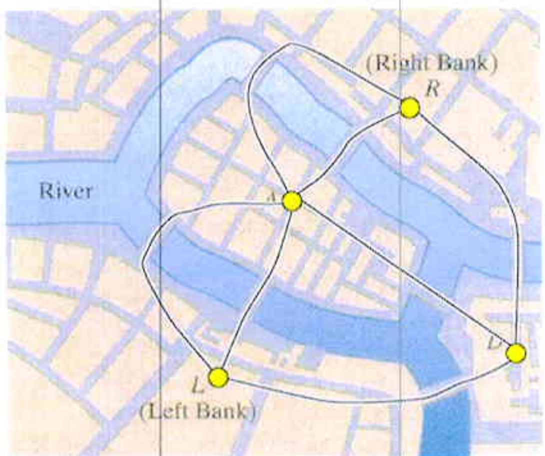
Solve the problem in mathematics

Translate the solution back into real life.

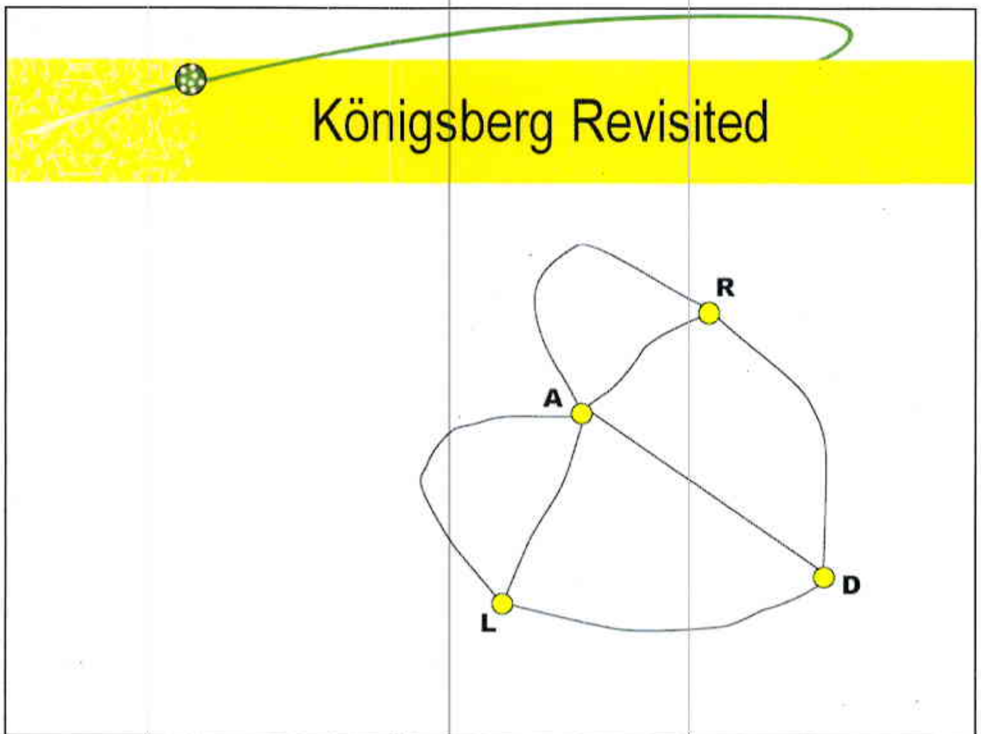
Look at Königsberg again. Many details are irrelevant to the problem: the width of the rivers, size of the land masses, the various streets, etc.

What really matters is surprisingly little: the relationships among land masses (banks and islands) and bridges. Which land masses are connected, and by how many bridges?

Königsberg Revisited



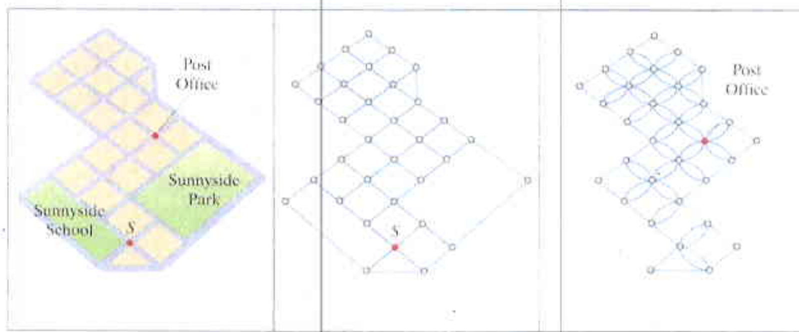
The 4 vertices represent the 4 land masses, and the edges represent the 7 bridges.



To win the A for the quarter, you need only find an Euler Circuit of this graph, and for the A on the next test, you only need to find an Euler path.

Euler's realization that the problem of walking bridges could be reduced to an abstract question about graphs was a major breakthrough in the history of mathematics.

Sunnyside Revisited



Let's return to Sunnyside.

For security, the problem is reduced to the center graph. Does the graph have an Euler path? An Euler circuit?

For the mail carrier, the problem is reduced to the right graph. Why are the graphs different?

Our next step is to begin to look at approaches to solving these problems.