



Plurality with Elimination

More ways to count....

Chapter 1b.ppt



Plurality-with-Elimination

- Also called "Instant Runoff"
- Step 1: Count the first-place votes, just as you would in the plurality method. If a candidate has a majority, then that candidate is the winner. Otherwise, eliminate the candidate with the *fewest* first-place votes.

We work our way up to a majority....





Round 1

| Votes | 14 | 10 | 8 | 4 | 1 | |
|-----------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------|
| 1 st | A | C | D | B | C | → A 14 |
| 2 nd | B | B | C | D | D | B 4 |
| 3 rd | C | D | B | C | B | C 11 |
| 4 th | D | A | A | A | A | D 8 |

How many voters? (37)

What is a majority? (19)

So no candidate has a majority, so we eliminate B who has the fewest 1st place votes ✂ [automatic] and move the rest up. Remember, we can do that with a preference schedule.





Round 2

Votes 14 10 8 4 1

| | | |
|-----------------|---|---------------|
| 1 st | A C D D C | → A 14 |
| 2 nd | C D C C D | C 11 |
| 3 rd | D A A A A | D 12 |

Still no majority. Whom do we eliminate? (C) ✂ [automatic] and move everybody up.





Round 3

Votes 14 10 8 4 1

| | | | | | | |
|-----------------|----------|----------|----------|----------|----------|---------------|
| 1 st | A | D | D | D | D | → A 14 |
| 2 nd | D | A | A | A | A | D 23 |

D clearly has a majority, and thus D is the winner.

Interestingly enough---





Interesting...

- That election, by the plurality method, went to A
- By the Borda Count method, it went to B
- But by the Plurality-with-Elimination method, it went to D

By plurality A won,

But by Borda, it went to B,

And by Plurality with Elimination, it went to D.

Let's look at another that we have seen before....





Note this example

| | Votes: 6 | 2 | 3 |
|-----------------|----------|----------|----------|
| 1 st | A | B | C |
| 2 nd | B | C | D |
| 3 rd | C | D | B |
| 4 th | D | A | A |

Remember: under plurality, A won, and under Borda, B won, and A was the Condorcet candidate.

But note that A not only has a plurality, but also a majority, so by Plurality with elimination, we are done.





Note this example

- In this case there are no rounds to go through: A has received a majority, and thus wins under plurality with elimination
- Therefore: The Plurality with Elimination method satisfies the majority criterion

Important: Plurality with Elimination satisfies the majority criterion.





Summer Olympics

- Lets consider another election:
 - Voting for the location of the 2020 Summer Olympics (Athens, Barcelona and Calgary), a straw poll is held, using plurality with elimination.

Often, when trying to settle something, a straw vote is held. It is not binding, but just lets everyone see where things are.





Olympics: Straw Poll 1

| | Votes: 7 | 8 | 10 | 4 |
|-----------------|----------|----------|----------|----------|
| 1 st | A | B | C | A |
| 2 nd | B | C | A | C |
| 3 rd | C | A | B | B |

Ok. How many voters? (29)

What's a majority? (15)

Whom do we eliminate? (Barcelona)





Olympics: Straw Poll 2

| | Votes: 7 | 8 | 10 | 4 |
|-----------------|----------|----------|----------|----------|
| 1 st | A | C | C | A |
| 2 nd | C | A | A | C |

So Calgary is the winner of the straw poll

And Calgary, with 18, is the winner.





Summer Olympics

- The straw poll was supposed to be secret, but word got out, and the voters in the last column (A-C-B) decided to switch their votes to vote for Calgary first, then Athens, so they, too, could vote for the winner...
 - Let's see what happens

The straw poll was supposed to be secret, but word got out, and the voters in the last column (A-C-B) decided to switch their first and second place votes, so they, too, could vote for the winner...

Let's see what happens...





Olympics: Straw Vote

| Votes: | 7 | 8 | 10 | 4 |
|-----------------|----------|----------|----------|----------|
| 1 st | A | B | C | A |
| 2 nd | B | C | A | C |
| 3 rd | C | A | B | B |

Here's the straw vote, so the last column people switch their votes...





Olympics: Final Vote 1

| | Votes: 7 | 8 | 10 | 4 |
|-----------------|----------|----------|----------|----------|
| 1 st | A | B | C | C |
| 2 nd | B | C | A | A |
| 3 rd | C | A | B | B |

... note that the last 2 columns are now identical, so they can be combined for 14 votes....





Olympics: Final Vote 1

| | Votes: 7 | 8 | 14 |
|-----------------|----------|----------|----------|
| 1 st | A | B | C |
| 2 nd | B | C | A |
| 3 rd | C | A | B |

No majority, so we eliminate Athens, with only 7 first place votes...





Olympics: Final Vote 2

| | Votes: 7 | 8 | 14 |
|-----------------|----------|----------|----------|
| 1 st | B | B | C |
| 2 nd | C | C | B |

And Barcelona is the winner!

And Barcelona wins.

WHAT? Whoa... how is that possible?

Let's recap.





Olympics: Straw Vote

| | Votes: 7 | 8 | 10 | 4 |
|-----------------|----------|----------|----------|----------|
| 1 st | A | B | C | A |
| 2 nd | B | C | A | C |
| 3 rd | C | A | B | B |

When the first vote was this, Calgary won...



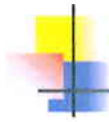


Olympics: Final Vote

| | Votes: 7 | 8 | 10 | 4 |
|-----------------|----------|----------|----------|----------|
| 1 st | A | B | C | C |
| 2 nd | B | C | A | A |
| 3 rd | C | A | B | B |

...but when Calgary had 2 additional 1st place votes, Barcelona won.





Summer Olympics

- How is this possible?
 - Calgary is the victim of a quirk in the plurality with elimination method:
 - It is possible to do *worse* by doing *better*!
 - This is an example of a violation of the monotonicity criterion.

How is this possible?

Calgary is the victim of a quirk in the plurality with elimination method:

It is possible to do *worse* by doing *better*!

This is an example of a violation of the *monotonicity criterion*.





The Monotonicity Criterion

- If candidate X is a winner of an election and, in a reelection, the only changes in the ballots are changes that favor X (and only X), then X should remain a winner of the election.

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It would also be easy to come up with an example that shows that plurality-with-elimination can also violate the Condorcet criteria.

In spite of this, plurality with elimination is becoming more widely used....





Plurality with Elimination

- Used by:
 - Olympic Committee
 - San Francisco municipal elections
 - Burlington, VT municipal elections
 - Other US cities in process of adopting this
 - Berkeley, CA; Ferndale, MI...
 - Members of House of Representatives in Australia.





Pairwise Comparisons

or
Copeland's Method

One more method...



So far...

- The methods we have considered so far can violate the Condorcet Criterion
 - Plurality
 - Borda Count
 - Plurality with Elimination
- New method of pairwise comparisons satisfies the Condorcet criterion

All the methods we have considered so far can violate the Condorcet Criterion

- Plurality
- Borda Count
- Plurality with Elimination

The new method of pairwise comparisons satisfies the Condorcet criterion

