

We'll look at this election	on again	
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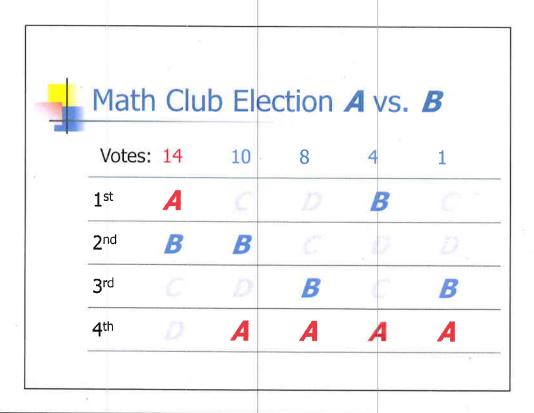


Math Club Election A vs. B

 We will consider only candidates A (red) and B (blue), and ignore all the rest.

And we'll start by considering only candidates A and B, and ignore the rest....





We notice that A is above B for 14 votes, and

B is above A for 10 + 8 + 4 + 1 or 23 votes, so...



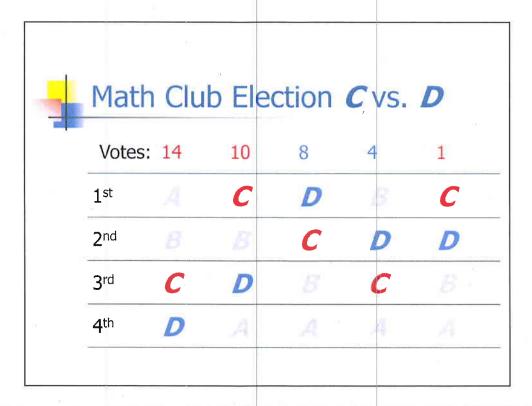
Math Club Election A vs. B

A vs B: 14 red votes to 23 blue votes, so B wins, and gets 1 point:

A	0
B	1
C	0
D	0

...we award B one point.

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Now we repeat the procedure, this time using only C and D.

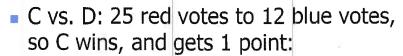
C is above D 14 + 10 + 1 = 25 times

D is above C 8 + 4 = 12 times

So....



Math Club Election Cvs. D



A	0
В	1
С	1
D	0

C wins and gets 1 point.



Math Club Election

- If we continue, comparing all possible combinations of two candidates (that's ${}_{4}C_{2} = 6$ comparisons), we find:
 - A vs. B: 14 to 23, B wins 1 point
 - A vs. C: 14 to 23, C wins 1 point
 - A vs. D: 14 to 23, D wins 1 point
 - B vs. C: 18 to 19, C wins 1 point
 - B vs. D: 28 to 9, B wins 1 point
 - C vs. D: 25 to 12, C wins 1 point

If we continue, doing the same thing with each possible pair, we get these results...





Math Club Election

So the final tally is:

A	0
В	2
C	3
D	1

and the winner is candidate C

So the final tally is shown here, and C is the winner.





Pairwise Comparisons

- A majority candidate automatically wins every pairwise comparison, so the majority criterion is satisfied.
- A Condorcet candidate wins every pairwise comparison by definition, so the *Condorcet criterion* is satisfied
- It can also be shown that this method satisfies the monotonicity criterion.

The method of pairwise comparisons is looking pretty good...

It satisfies the majority criterion, the Condorcet criterion and the monotonicity criterion.

But the method is not perfect...





Pairwise Comparions

- Unfortunately, the method can violate a fourth criterion...
- Consider the NFL Draft, where the Los Angeles LAXers will get the number one pick. The list is narrowed to 5 players (Allen, Byers, Castillo, Dixon, and Evans) and the method of pairwise comparisons is used.

There is a 4th criterion, and this method can violate it.

Let's look at a hypothetical NFL Draft...





So 22 preference ballots are cast and the preference schedule looks like this....

If we do all the pairwise comparisons (how many are there? $-{}_{5}C_{2} = 10$), we find....





NFL Draft

A/B	7	15	В
A/C	16	6	Α
A/D	13	9	Α
A/E	18	4	Α
В/С	10	12	С

B/D	11	11	tie
В/Е	14	8	В
C/D	12	10	С
C/E	10	12	Е
D/E	18	4	D

...the following. Note that for the B/D comparison, which is a tie, each player receives ½ point.

So we have...



NFL Draft

- So it looks like Allen
 (A) is the winner.
- But: it is discovered right before the draft that Castillo (C) will not be playing pro ball.
- Shouldn't make a difference, right?

Α	3	
В	21/2	
С	2	
D	11/2	
E	1	

And Allen is the winner.

But,

It is discovered that Castillo will not play at all. Since he was third, it shouldn't make a difference, but...



If we drop C out and slide the others up			
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*			



And perform the process all over	r again		
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9		, s	0
9			



NFL Draft

- If we go through the calculations this time, we find that Byers (**B**) is the winner!
- Candidate C was irrelevant to the decision, but his removal changes the results.

Α	2
В	21/2
D	11/2
Е	0

... it turns out that Beyers is the winner. C was irrelevant to the results, but his removal changed them, nevertheless...





Independence-of-Irrelevant Alternatives Criterion

If candidate X is a winner of an election and in a recount one of the nonwinning candidates is removed from the ballots, then X should still be a winner of the election.

This violates the 4th criterion, the Independence of Irrelevant Alternatives Criterion. We often call this "IIA"

If candidate X is a winner of an election and in a recount one of the non-winning candidates is removed from the ballots, then X should still be a winner of the election.

