Math Club Election (again)

Votes:	14	10	8	4	1
1st	***A***	***C***	***D***	***B***	***C***
2nd	***B***	***B***	***C***	***D***	***D***
3rd	***C***	***D***	***B***	***C***	***B***
4th	***D***	***A***	***A***	***A***	***A***

We'll look at this election again…

Math Club Election *A* vs. *B*

- We will consider only candidates *A* (red) and *B* (blue), and ignore all the rest.

And we'll start by considering only candidates A and B, and ignore the rest….

⌘

Math Club Election *A* vs. *B*

Votes:	14	10	8	4	1
1st	**A**	C	D	**B**	C
2nd	**B**	**B**	C	D	D
3rd	C	D	**B**	C	**B**
4th	D	**A**	**A**	**A**	**A**

We notice that A is above B for 14 votes, and

B is above A for 10 + 8 + 4 + 1 or 23 votes, so…

Math Club Election ***A*** vs. ***B***

- A vs B: 14 red votes to 23 blue votes, so B wins, and gets 1 point:

A	**0**
B	**1**
C	**0**
D	**0**

…we award B one point.

⌘

Math Club Election *C* vs. *D*

Votes:	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

Now we repeat the procedure, this time using only C and D.

C is above D 14 + 10 +1 = 25 times

D is above C 8 + 4 = 12 times

So….

Math Club Election *C* vs. *D*

- C vs. D: 25 red votes to 12 blue votes, so C wins, and gets 1 point:

A	**0**
B	**1**
C	**1**
D	**0**

C wins and gets 1 point.

⌘

Math Club Election

- If we continue, comparing all possible combinations of two candidates (that's ${}_4C_2 = 6$ comparisons), we find:
  - A vs. B: 14 to 23, B wins 1 point
  - A vs. C: 14 to 23, C wins 1 point
  - A vs. D: 14 to 23, D wins 1 point
  - B vs. C: 18 to 19, C wins 1 point
  - B vs. D: 28 to 9, B wins 1 point
  - C vs. D: 25 to 12, C wins 1 point

If we continue, doing the same thing with each possible pair, we get these results…

⌘

Math Club Election

- So the final tally is:

A	0
B	2
C	3
D	1

- and the winner is candidate C

So the final tally is shown here, and C is the winner.

Pairwise Comparisons

- A majority candidate automatically wins every pairwise comparison, so the *majority criterion* is satisfied.
- A Condorcet candidate wins every pairwise comparison by definition, so the *Condorcet criterion* is satisfied
- It can also be shown that this method satisfies the *monotonicity criterion.*

The method of pairwise comparisons is looking pretty good…

It satisfies the majority criterion, the Condorcet criterion and the monotonicity criterion.

But the method is not perfect…

Pairwise Comparions

- Unfortunately, the method can violate a fourth criterion…
- Consider the NFL Draft, where the Los Angeles LAXers will get the number one pick. The list is narrowed to 5 players (Allen, Byers, Castillo, Dixon, and Evans) and the method of pairwise comparisons is used.

There is a 4th criterion, and this method can violate it.

Let's look at a hypothetical NFL Draft…

NFL Draft

Vote	2	6	4	1	1	4	4
1st	*A*	*B*	*B*	*C*	*C*	*D*	*E*
2nd	*D*	*A*	*A*	*B*	*D*	*A*	*C*
3rd	*C*	*C*	*D*	*A*	*A*	*E*	*D*
4th	*B*	*D*	*E*	*D*	*B*	*C*	*B*
5th	*E*	*E*	*C*	*E*	*E*	*B*	*A*

So 22 preference ballots are cast and the preference schedule looks like this….

If we do all the pairwise comparisons (how many are there? – ${}_5C_2 = 10$), we find….

NFL Draft

A/B	7	15	B
A/C	16	6	A
A/D	13	9	A
A/E	18	4	A
B/C	10	12	C

B/D	11	11	tie
B/E	14	8	B
C/D	12	10	C
C/E	10	12	E
D/E	18	4	D

…the following. Note that for the B/D comparison, which is a tie, each player receives ½ point.

So we have…

NFL Draft

- So it looks like Allen (***A***) is the winner.
- But: it is discovered right before the draft that Castillo (***C***) will not be playing pro ball.
- Shouldn't make a difference, right?

A	3
B	2½
C	2
D	1½
E	1

And Allen is the winner.

But,

It is discovered that Castillo will not play at all. Since he was third, it shouldn't make a difference, but…

NFL Draft – Round 2

Vote	2	6	4	1	1	4	4
1st	***A***	***B***	***B***	~~C~~	~~C~~	***D***	***E***
2nd	***D***	***A***	***A***	***B***	***D***	***A***	~~C~~
3rd	~~C~~	~~C~~	***D***	***A***	***A***	***E***	***D***
4th	***B***	***D***	***E***	***D***	***B***	~~C~~	***B***
5th	***E***	***E***	~~C~~	***E***	***E***	***B***	***A***

If we drop C out and slide the others up…

NFL Draft – Round 2

Vote	2	6	4	1	1	4	4
1st	*A*	*B*	*B*	*B*	*D*	*D*	*E*
2nd	*D*	*A*	*A*	*A*	*A*	*A*	*D*
3rd	*B*	*D*	*D*	*D*	*B*	*E*	*B*
4th	*E*	*E*	*E*	*E*	*E*	*B*	*A*

And perform the process all over again…

NFL Draft

- If we go through the calculations this time, we find that Byers (***B***) is the winner!
- Candidate ***C*** was irrelevant to the decision, but his removal changes the results.

A	2
B	2½
D	1½
E	0

… it turns out that Beyers is the winner. C was irrelevant to the results, but his removal changed them, nevertheless…

Independence-of-Irrelevant Alternatives Criterion

- If candidate *X* is a winner of an election and in a recount one of the non-winning candidates is removed from the ballots, then *X* should still be a winner of the election.

This violates the 4th criterion, the Independence of Irrelevant Alternatives Criterion. We often call this "IIA"

If candidate *X* is a winner of an election and in a recount one of the non-winning candidates is removed from the ballots, then *X* should still be a winner of the election.