An Interesting Election

Votes:	4	2	6	3
1st	***A***	***B***	***C***	***B***
2nd	***B***	***C***	***A***	***A***
3rd	***C***	***A***	***B***	***C***

Here is an interesting election….

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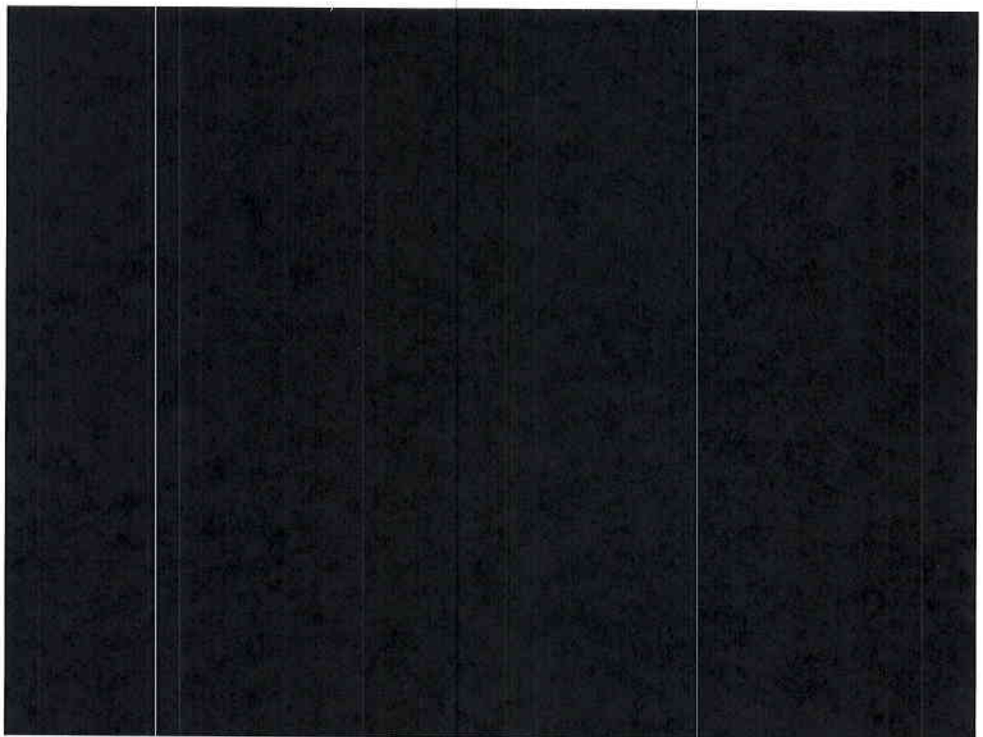
An Interesting Election

- *Be sure you can do the math for the following:*
  - Plurality gives the election to *C*
  - Borda Count gives the election to *A*
  - Plurality with Elimination gives the election to *B*
  - Pairwise comparison results in a 3-way tie!

Try this tonight. Be certain you can reproduce all these results….

Time Out

For some boardwork...



How many pairwise comparisons among n candidates? ${}_nC_2$

Story about Little Karl Friedrich Gauss….

Rankings

We'll talk about rankings now. For example, our club will have an election, and the winner will be president, the 2nd will be VP and the 3rd will be treasurer.

Rankings One

- If we need to rank all the candidates in an election, there are two ways to accomplish this.
- The first is a direct extension of the counting methods we've used

Two ways to do this…

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Rankings: Extended Plurality

Votes:	14	10	8	4	1
1st	***A***	***C***	***D***	***B***	***C***
2nd	***B***	***B***	***C***	***D***	***D***
3rd	***C***	***D***	***B***	***C***	***B***
4th	***D***	***A***	***A***	***A***	***A***

Look at only 1st place votes in this election. We have…

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Rankings: Extended Plurality

A: 14   C: 11   D: 8   B: 4

Votes:	14	10	8	4	1
1st	A	C	D	B	C
2nd	B	B	C	D	D
3rd	C	D	B	C	B
4th	D	A	A	A	A

A with 14 is 1st

C with 11 is second

D with 8 is third

And B with 4 is 4th

We call this method Ranking by Extended Plurality.

Rankings: Extended Borda

Votes:	14	10	8	4	1
1st	***A***	***C***	***D***	***B***	***C***
2nd	***B***	***B***	***C***	***D***	***D***
3rd	***C***	***D***	***B***	***C***	***B***
4th	***D***	***A***	***A***	***A***	***A***

We can also extend the Borda Count method. We go through all the Borda steps as usual…

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Rankings: Extended Borda

B: 106   C: 104   D: 81   A: 79

Votes:	14	10	8	4	1
1st	**A**	**C**	**D**	**B**	**C**
2nd	**B**	**B**	**C**	**D**	**D**
3rd	**C**	**D**	**B**	**C**	**B**
4th	**D**	**A**	**A**	**A**	**A**

…and find that:

B has 106 points and is 1st

C has 104 and is 2nd

D has 81 and is 3rd

And A has 70 points and is 4th

Rankings: Extended Plurality with Elimination

- We rank the candidates in the reverse order in which they are eliminated:

Rank	Candidate	Eliminated
1st	D	*(not)*
2nd	A	Round 3
3rd	C	Round 2
4th	B	Round 1

To extend plurality with elimination, we simply keep track of the order in which candidates are eliminated, and that is the reverse of the final ranking.

Rankings: Extended Pairwise Comparisons

- We use the total number of points in their comparisons with other candidates:

Rank	Candidate	Points
1st	C	3
2nd	B	2
3rd	D	1
4th	A	0

For extended pairwise, we have a set of points that we can use to arrive at a ranking.

Rankings: Election Results

Votes:	14	10	8	4	1
1st	***A***	***C***	***D***	***B***	***C***
2nd	***B***	***B***	***C***	***D***	***D***
3rd	***C***	***D***	***B***	***C***	***B***
4th	***D***	***A***	***A***	***A***	***A***

Here's the election

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Extended Methods Summary

Method	Ranking 1st	2nd	3rd	4th
Extended plurality	A	C	D	B
Extended Borda count	B	C	D	A
Extended Plurality/Elim	D	A	C	B
Extended pairwise	C	B	D	A

And here are the rankings by the 4 different methods. Make certain you can do the math to arrive at this table…

Rankings Two

- The second method of ranking all the candidates is called *Recursive Ranking.*
- The normal method is used to determine the winner: that candidate is ranked 1st.
- That candidate is then eliminated, and the process repeated to find the 2nd place candidate, and so forth…

The second method of ranking all the candidates is called *Recursive Ranking*.

The normal method is used to determine the winner: that candidate is ranked 1st.

That candidate is then eliminated, and the process starts all over again to find the 2nd place candidate, and so forth…

Recursive Ranking

- This can be done with any of the methods. Let's start looking at it with plurality.

Here we go…

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Rankings: Recursive Plurality

Step 1: A is 1st with 14. Now eliminate A

Votes:	14	10	8	4	1
1st	*A*	*C*	*D*	*B*	*C*
2nd	*B*	*B*	*C*	*D*	*D*
3rd	*C*	*D*	*B*	*C*	*B*
4th	*D*	*A*	*A*	*A*	*A*

Rankings: Recursive Plurality

Step 2: B wins with 18. Now eliminate B

Votes:	14	10	8	4	1
1st	***B***	***C***	***D***	***B***	***C***
2nd	***C***	***B***	***C***	***D***	***D***
3rd	***D***	***D***	***B***	***C***	***B***

Rankings: Recursive Plurality

Step 3: C wins with 25. D is last.

Votes:	14	10	8	4	1
1st	C	C	D	D	C
2nd	D	D	C	C	D

Rankings: Recursive Plurality

Rank	Candidate
1st	A
2nd	B
3rd	C
4th	D

Rankings: Summary

- If we compare the results of a recursive method with the results of the same method extended, we often find very real differences. So which is better?
  - In real life, the extended methods are almost always used.
  - BUT, there really is no simple answer to the question.

Voting Summary

- Which method to use?
- We've discussed four fairness criteria:
  - The majority criterion
  - The Condorcet Criterion
  - The Monotonicity Criterion
  - The Independence-of-Irrelevant-Alternatives Criterion.

Which method to use? A big question..

We have discussed 4 fairness criteria:

- The majority criteria
- The Condorcet criteria
- The monotonicity Criteria
- The Independence of Irrelevant Alternatives criteria.

[make certain they know each and when it applies]

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Voting Summary

- A fair voting method should satisfy all of those criteria.
- None that we've looked at do

It seems reasonable that that there should be a way to satisfy all four criteria.

We haven't seen it yet in here…

Criteria Violations

Method	Violations: Major-ity	Con-dorcet	Mono-tonic.	IIA
Plurality		√		√
Borda Count	√	√		√
Plural w/Elim		√	√	√
Pairwise Comp				√

Here is a summary of the kinds of violations that can occur (note: not necessarily do occur).

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So what method is best?

- For democratic elections involving three or more candidates, there is *no voting method* that satisfies the four criteria.
- This is **Arrow's Imposibility Theorem**, proven by Kenneth J. Arrow in 1949: "*It is mathematically impossible for a democratic voting method to satisfy all of the fairness criteria.*"

There is no *best* voting method.

This was proved by Kenneth Arrow in 1949.