

Instructions: Use combinations to count the number of things in each scenario. Write the permutation notation, express in terms of factorials, reduce, and then use the calculator if needed.

1. A deck of cards has 52 cards in it. You draw 5. How many 5-card hands are there?

$$52C5 = \binom{52}{5} = 2,598,960$$

2. Suppose a lottery drawing has 39 balls in the machine. 7 balls are chosen. How many winning combinations are there?

if order does not matter: $39C7 = \binom{39}{7} = 15,380,937$

3. How many different ways can you choose two of the numbers 1, 2, 3, and 4? What are they?

$$\binom{4}{2} = 4C2 = 6$$

$$\frac{4!}{2!2!} = \frac{24}{4} = 6$$

4. If a math department has 17 faculty members available to serve on advisory committee. How many ways can 6 committee members be chosen?

$$17C6 = \binom{17}{6} = \frac{17!}{6!11!} = \frac{17 \cdot \overset{4}{\cancel{16}} \cdot \cancel{15} \cdot 14 \cdot 13 \cdot \cancel{12}}{\cancel{6} \cdot \cancel{8} \cdot 4 \cdot \cancel{3} \cdot 2} = 12,376$$

5. How many different ways can we choose three of the letters A, B, C, D, E? Can you list them all?

$$5C3 = \binom{5}{3} = \frac{5!}{3!2!} = \frac{5 \cdot 4}{2} = 10$$

ABC ABD ABE ACD ACE ADE
BCD BCE BDE CDE

6. A class has 32 students in it that need to be divided into 8 working groups of 4. How many different ways can this be done? [Hint: choose the first group of 4, figure out many there are, then with the remaining students, do the next group, and so forth.]

$$\begin{aligned} & (32C4)(28C4)(24C4)(20C4)(16C4)(12C4)(8C4)(4C4) \\ & = 2.39 \times 10^{24} \end{aligned}$$

7. How many different ways can you get 6 heads if you flip a coin 15 times?

$${}_{15}C_6 = \binom{15}{6} = 5005$$

8. There are 5 door prizes given out at a raffle. Suppose there are 300 people who bought raffle tickets. How many different ways can the door prizes be given away?

$${}_{300}C_5 = \binom{300}{5} = 1.96 \times 10^{10}$$

9. Suppose there are 30 marbles in a bowl each of a different colour. How many different ways are there to grab a handful of six marbles?

$${}_{30}C_6 = \binom{30}{6} = 593,775$$

10. There are 4 voters in an election. How many different ways are there for all 4 voters to agree on a motion? How many ways are there for three voters to agree on a motion? How many ways are there for two voters to agree on a motion? How many ways are there for one voter to agree to a motion? How many total ways are there to vote on a motion where at least one person votes yes?

$${}_{4}C_4 + {}_{4}C_3 + {}_{4}C_2 + {}_{4}C_1 = 1 + 4 + 6 + 4 = 15$$

11. Suppose a family has 6 children. How many different ways are there for the family to have three boys and three girls?

$${}_{6}C_3 = \binom{6}{3} = 20$$