Math 1151, Curve Sketching Activity, Fall 2014 Name KEY

Instructions: For each of the functions below, find the first and second derivatives, any critical points and points of inflection, all regions where the graph is increasing and decreasing, and the concavity of the graph. Use this information to sketch the graphs of the function.

1. $y = \frac{x^2}{x^2+3}$

$$y' = \frac{2x(x^2+3) - 2x\cdot x^2}{(x^2+3)^2} = \frac{2x^3+6x-2x^3}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2} = 0$$

critical
$x=0$

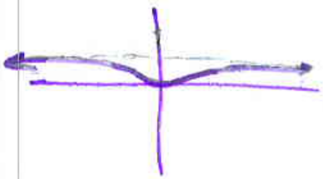
$$y'' = \frac{6(x^2+3)^2 - 2(x^2+3)(2x)6x}{(x^2+3)^4} = \frac{\cancel{(x^2+3)}(6(x^2+3) - 24x^2)}{(x^2+3)^{\cancel{4}3}} = \frac{6x^2+18 \ -24}{(x^2+3)^3}$$

$$= \frac{-18x^2+18}{(x^2+3)^3} = \frac{-18(x^2-1)}{(x^2+3)^3}$$

$x=1$, $x=-1$ inflection

$f'(x) \quad \overset{-}{\longleftarrow}\underset{0}{|}\overset{+}{\longrightarrow}$

$f''(x) \quad \overset{-}{\longleftarrow}\underset{-1}{|}\overset{+}{—}\underset{1}{|}\overset{-}{\longrightarrow}$



2. $y = x\sqrt{4-x}$ $= x(4-x)^{1/2}$

$$y' = \sqrt{4-x} + \frac{x(1/2)(-1)}{\sqrt{4-x}} = 0$$

$\left[\sqrt{4-x} = \frac{\frac{1}{2}x}{\sqrt{4-x}}\right]\sqrt{4-x} \Rightarrow 4-x = \frac{1}{2}x$

$4 = \frac{3}{2}x$

$x = 8/3$
critical

$$y'' = \frac{1}{2}(-1)\frac{1}{\sqrt{4-x}} - \frac{1}{2}\cdot\frac{1}{\sqrt{4-x}} - \frac{1}{2}x\cdot(-1/2)(-1)(4-x)^{3/2}$$

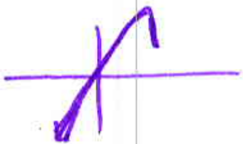
$$= \left[\frac{-1}{\sqrt{4-x}} - \frac{x}{4\sqrt{(4-x)^3}} = 0\right]\sqrt{(4-x)^3}$$

$-(4-x) - \frac{x}{4} = 0 \Rightarrow -4+x-\frac{1}{4}x = 0$

$\frac{3}{4}x = 4 \Rightarrow x = \frac{16}{3}$
inflecti

$f'(x) \quad \overset{+}{\longleftarrow}\underset{8/3}{|}\overset{-}{\longrightarrow}$

$\overset{-}{\longleftarrow}\underset{16/3}{|}\overset{+}{\longrightarrow}$



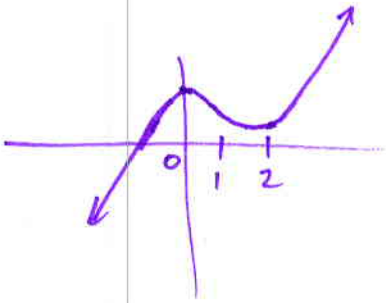
3. $x^3 - 3x^2 + 3 = y$

$y' = 3x^2 - 6x = 0 \quad 3x(x-2) = 0 \qquad x=0, x=2$

$y'' = 6x - 6 = 0 \quad 6(x-1) = 0 \quad x=1$

$f'(x) \quad \overset{+}{\longleftarrow}\underset{0}{|}\overset{-}{—}\underset{2}{|}\overset{+}{\longrightarrow}$

$f''(x) \quad \overset{-}{\longleftarrow}\underset{1}{|}\overset{+}{\longrightarrow}$

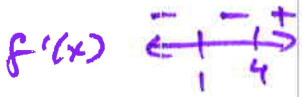


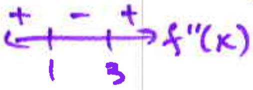
4. $y = x^4 - 8x^3 + 18x^2 - 16x + 5$

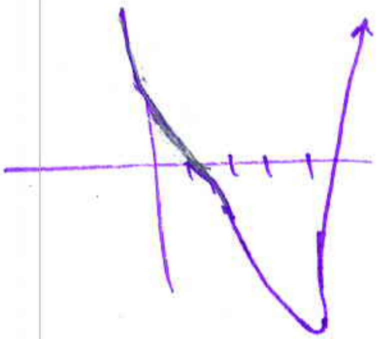
$y' = 4x^3 - 24x^2 + 36x - 16 = 4(x^3 - 6x^2 + 9x - 4) = 0 \quad x=1, x=4$

$y'' = 12x^2 - 48x + 36 = 12(x^2 - 4x + 3) = 0$

$(x-3)(x-1) = 0 \quad x = 1, 3$





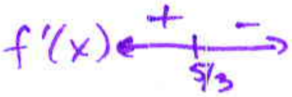


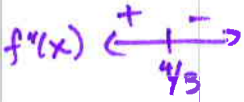
5. $y = e^{3x}(2 - x)$

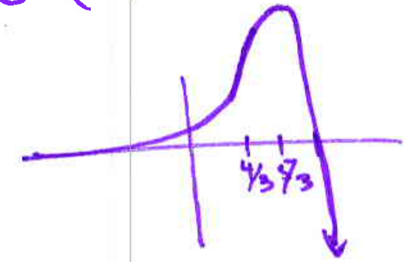
$y' = 3e^{3x}(2-x) + e^{3x}(-1)$

$e^{3x}(\underset{-3x+5}{6-3x-1}) = 0 \quad 3x = 5 \quad x = 5/3$

$y'' = 3e^{3x}(-3x+5) + e^{3x}(-3) = e^{3x}(-9x+15-3) = 0 \quad 9x = 12 \quad x = 4/3$







6. $y = 7\arctan(x + 1) - \ln(x^2 + 2x + 2)$

$y' = \frac{7}{1+(x+1)^2} - \frac{2x+2}{x^2+2x+2} = 0 \qquad \frac{-2x+5}{1+(x+1)^2} = 0 \Rightarrow 2x = 5 \Rightarrow x = 5/2$

$y'' = \frac{(-2)(x^2+2x+2) - (2x+2)(-2x+5)}{(x^2+2x+2)^2} = \frac{-2x^2 - 4x - 4 + 4x^2 - 10x + 4x - 10}{(x^2+2x+2)^2}$

$= \frac{2x^2 - 10x - 16}{(x^2+2x+2)^2} = \frac{2(x^2-5x-8)}{(x^2+2x+2)^2} = 0 \qquad \frac{5 \pm \sqrt{25-4(1)(-8)}}{2} = \frac{5 \pm \sqrt{57}}{2}$