Math 1151, Graphs of Derivatives Activity, Fall 2014 Name KEY

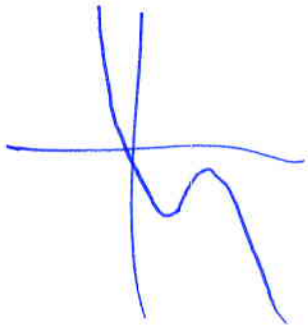
Instructions: For each of the functions below, find $f'(x) and f''(x)$. Graph the original function and its derivatives. Note any critical points and points of inflection.

1. $f(x) = -x^3 + 6x^2 - 9x - 1$

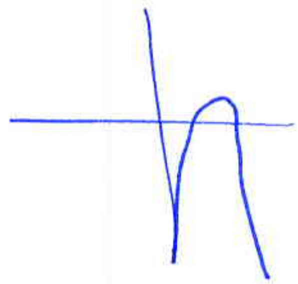
$f'(x) = -3x^2 + 12x - 9$

$f''(x) = -6x + 12$

f(x)



f'(x)



f''(x)

critical points: $x^2 - 4x + 3 = 0$ $(x-1)(x-3) = 0$ $x = 1, x = 3$

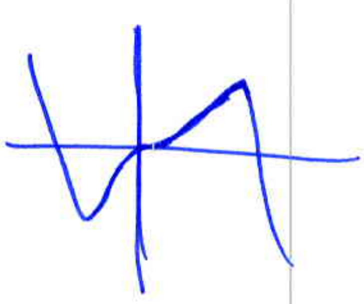
inflection points $x = 2$

2. $f(x) = \frac{1}{270}(-3x^5 + 40x^3 + 135x)$

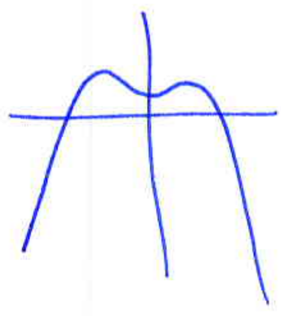
$f'(x) = \frac{1}{270}(-15x^4 + 120x^2 + 135)$

$f''(x) = \frac{1}{270}(-60x^3 + 240x)$

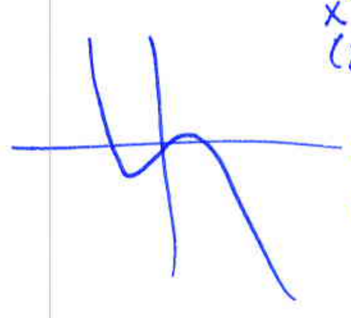
f(x)



f'(x)



f''(x)



critical points

$x^4 - 8x^2 - 9$

$(x^2 - 9)(x^2 + 1)$

$x = \pm 3$

inflection points

$x^3 - 4x = 0$

$x = 0, x = \pm 2$

3. $f(x) = x + 2\csc(x)$

$f'(x) = 1 - 2\csc x \cot x$

$f''(x) = +2\csc x \cot^2 x + 2\csc^3 x$

critical points

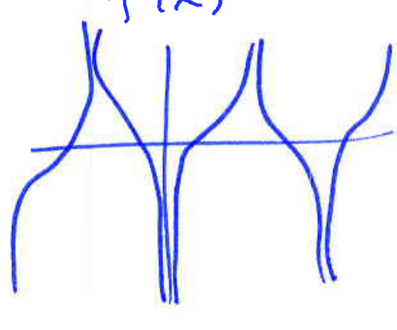
$\frac{1}{2} = \csc x \cot x$

$2 = \sin x \tan x$

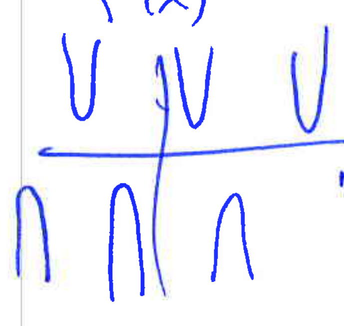
$x = \pm\sqrt{2}$ and others

f(x)

f'(x)



f''(x)



inflection points

$\csc x(\cot^2 x + \csc^2 x) = 0$

$\csc x$ never 0

$\cot^2 x = -\csc^2 x$ never 0

Instructions: For each problem below, the graphs of $f'(x)$ and $f''(x)$ are shown. What can you say about the graph of the original function? Note any critical points, increasing or decreasing intervals, inflection points and concavity. Sketch a graph consistent with these features.

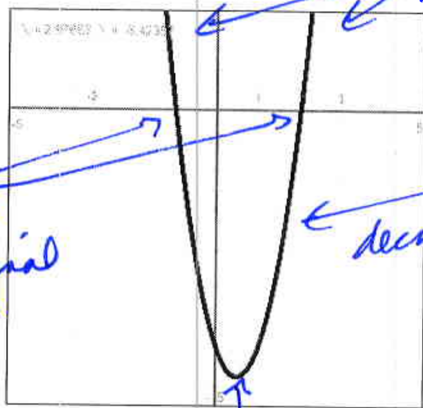
increasing

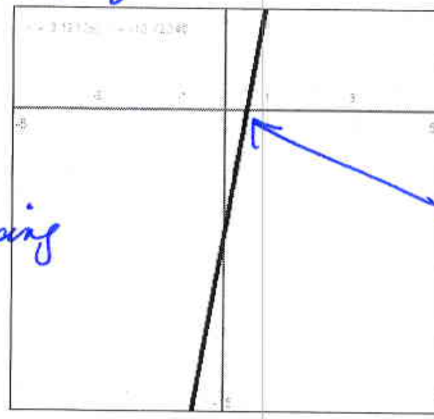
turning point of original graph

decreasing

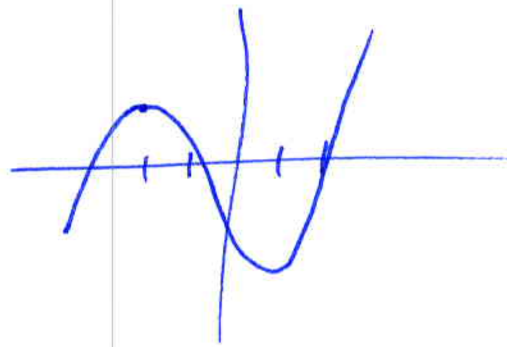
0 of $f''(x)$ is inflection pnt.

4.



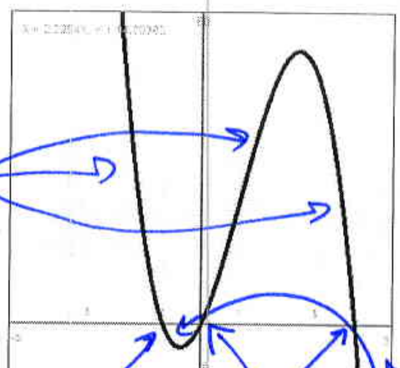


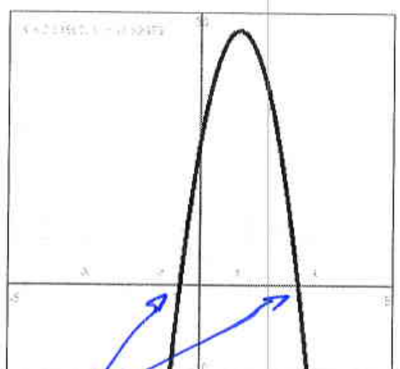
Critical point of $f'(x)$ is inflection pt



increasing

5.

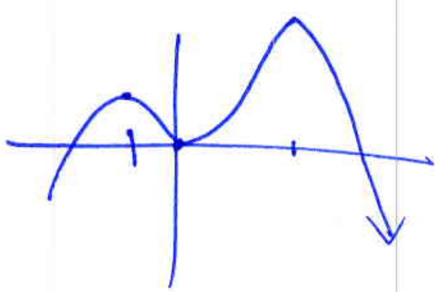




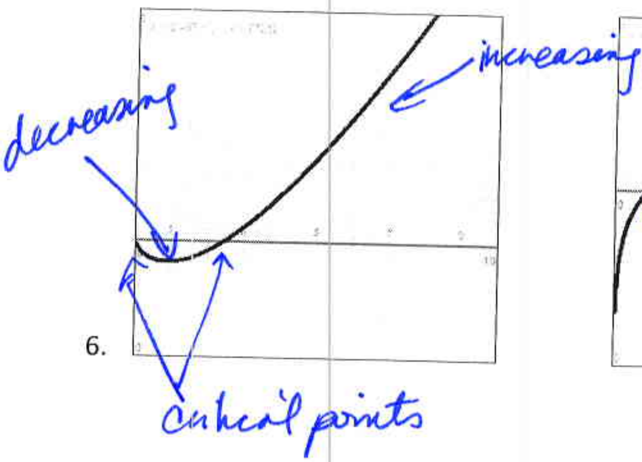
Critical point

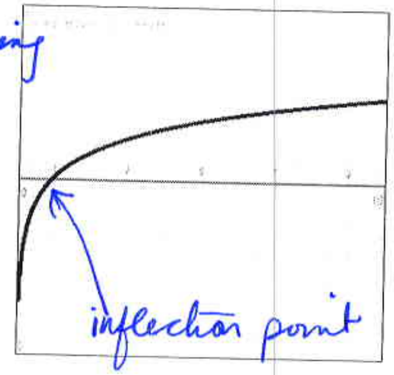
decreasing

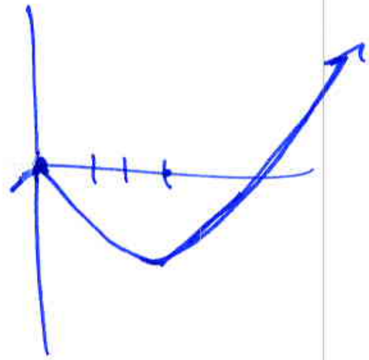
inflection point



6.







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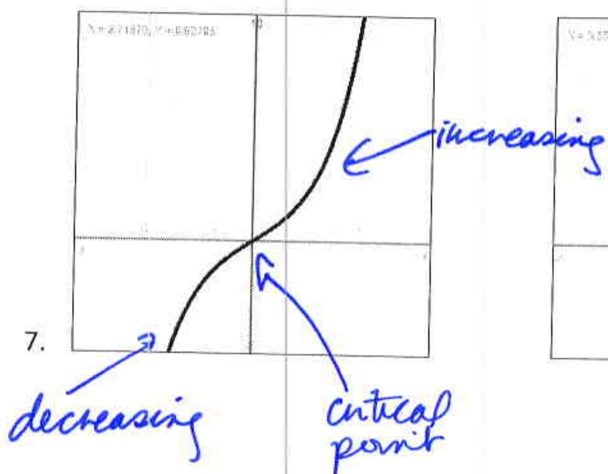
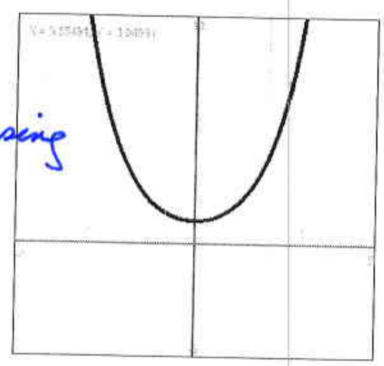
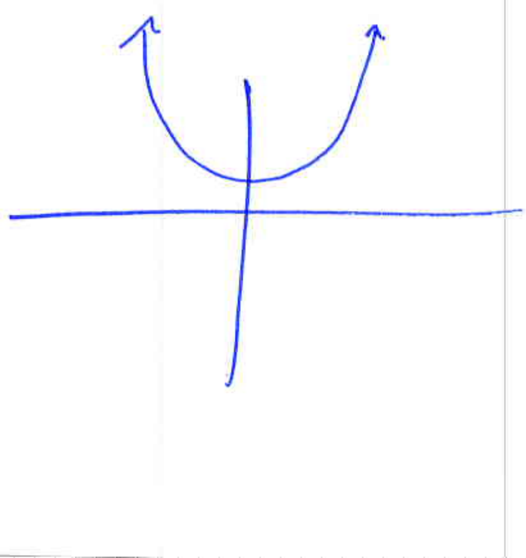




no inflection points



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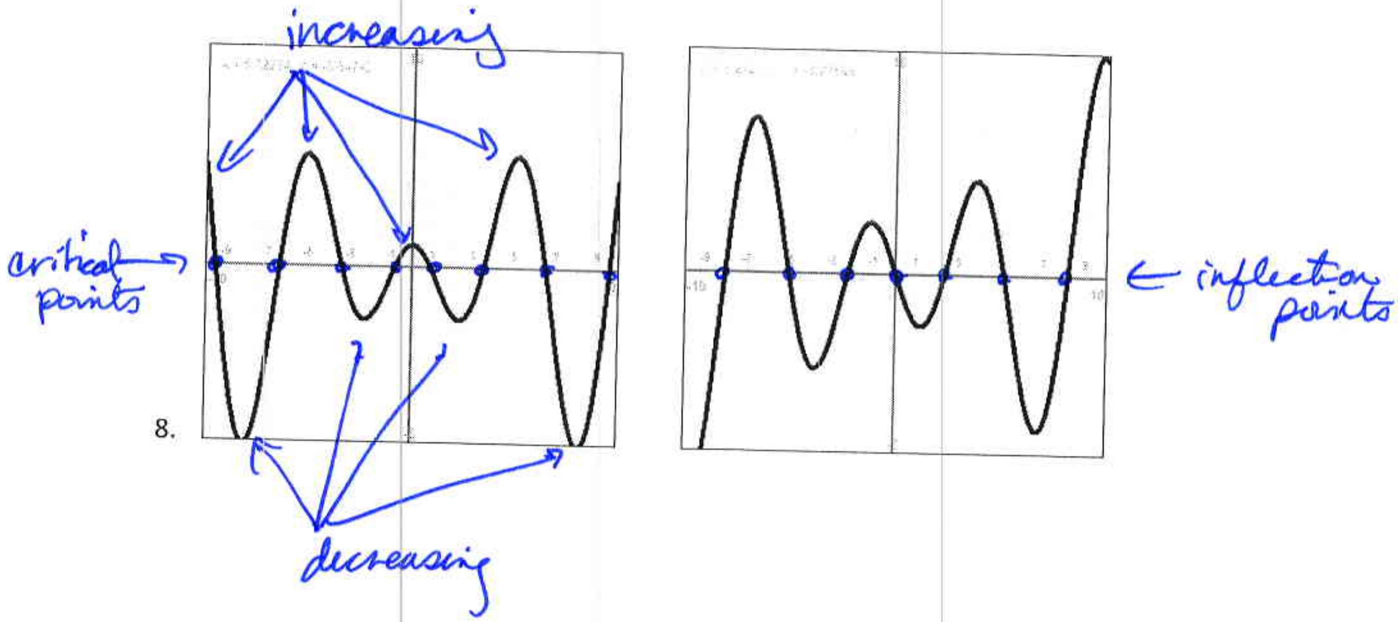
increasing
critical points
decreasing
inflection points