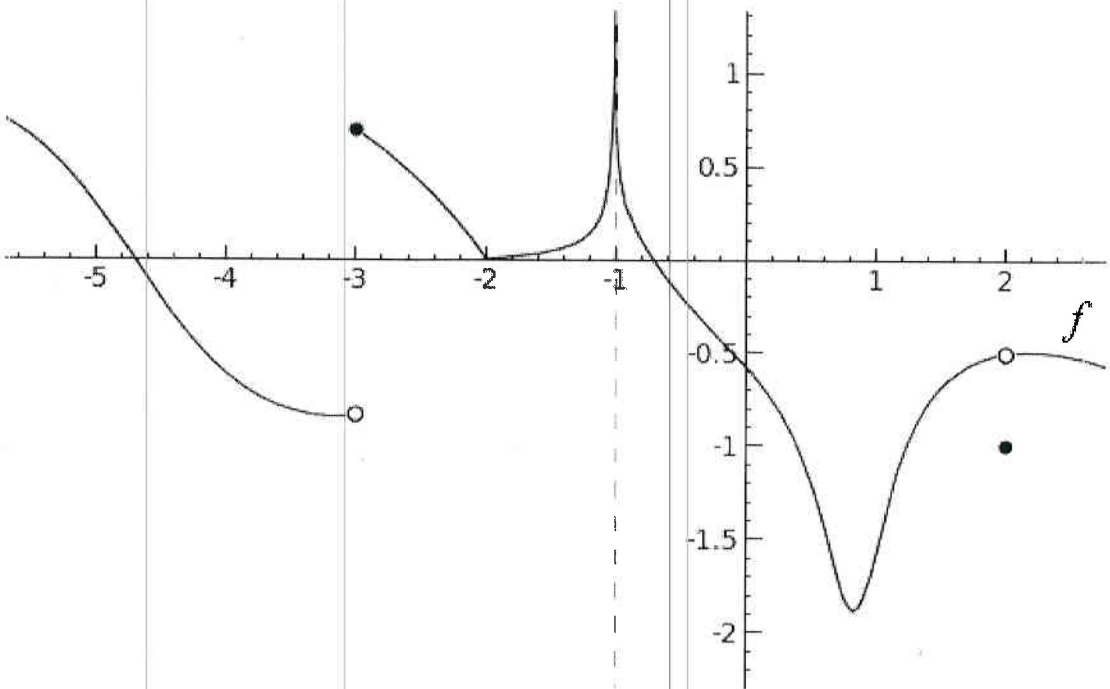
Math 1151, Exam #1, Fall 2014 Name KEY

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Shown below is the graph of $f(x)$. Find the limit at the indicated values. (2 points each)



a. $\lim_{x\to-2} f(x) = 0$

b. $\lim_{x\to-3^-} f(x) = -1$ (approx)

c. $\lim_{x\to-1} f(x) = \infty$

d. $\lim_{x\to2} f(x) = -1/2$

2. Use properties of limits and the fact that $\lim_{x\to1} f(x) = 4$, $\lim_{x\to1} g(x) = 7$. (2 points each)

a. $\lim_{x\to1} \frac{f(x)}{g(x)} = \frac{4}{7}$

b. $\lim_{x\to1} \frac{\sqrt{f(x)}}{g^2(x)} = \frac{\sqrt{4}}{49} = \frac{2}{49}$

c. $\lim_{x\to1} 4f(x) = 4(4) = 16$

d. $\lim_{x\to1} f(x)g(x) + 3 = 4(7)+3 = 31$

3. Find the limit algebraically of $\lim_{h\to0} \frac{\sqrt{16+h}-4}{h}$. (5 points)

$$\lim_{h\to0} \frac{(\sqrt{16+h}-4)(\sqrt{16+h}+4)}{h(\sqrt{16+h}+4)} = \lim_{h\to0} \frac{\cancel{16}+\cancel{h}-\cancel{16}}{\cancel{h}(\sqrt{16+h}+4)} = \lim_{h\to0} \frac{1}{\sqrt{16+h}+4} = \frac{1}{8}$$

4. Find the limit algebraically of $\lim_{x\to 0}\frac{\tan(5x)}{x}$. (5 points)

$$\lim_{x\to 0}\frac{5\cdot\sin 5x}{5\,x\cdot\cos 5x} = \left(\lim_{x\to 0}\frac{\sin 5x}{5x}\right)\cdot\left(\lim_{x\to 0}\frac{5}{\cos 5x}\right) = (1)\cdot\frac{5}{1} = 5$$

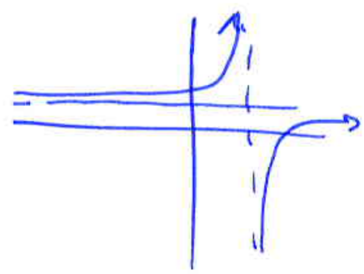
5. Find any and all asymptotes of the graph of $f(x)=\frac{x^2-9x+14}{x^2-5x+6}$. Explain how these asymptotes relate to limits. (It may be helpful to sketch the graph of the function.) (10 points)

possible vertical asymptotes $x^2-5x+6=0$ $(x-2)(x-3)=0$ $x=2, x=3$

horizontal asymptotes $\frac{x^2}{x^2}=1$ $\lim_{x\to\infty}\frac{x^2-9x+14}{x^2-5x+6}=1$

$$\lim_{x\to 2}\frac{x^2-9x+14}{x^2-5x+6} = \lim_{x\to 2}\frac{\cancel{(x-2)}(x-7)}{\cancel{(x-2)}(x-3)} = \frac{2-7}{2-3} = \frac{-5}{-1} = 5$$

$x=2$ is a hole

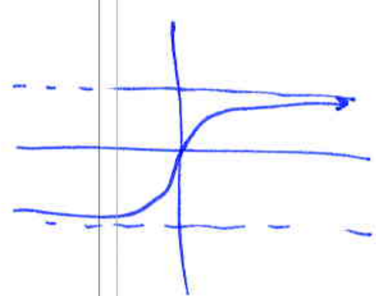


$$\lim_{x\to 3}\frac{x^2-9x+14}{x^2-5x+6} = \lim_{x\to 3}\frac{x-7}{x-3} = \text{DNE}$$

$\lim_{x\to 3^-} f(x) = \infty$ $\lim_{x\to 3^+} f(x) = -\infty$

6. Evaluate $\lim_{x\to\infty}\tan^{-1}(x)$. Give an exact answer, not a decimal approximation. (5 points)

$$\lim_{x\to\infty}\tan^{-1}(x) = \pi/2$$



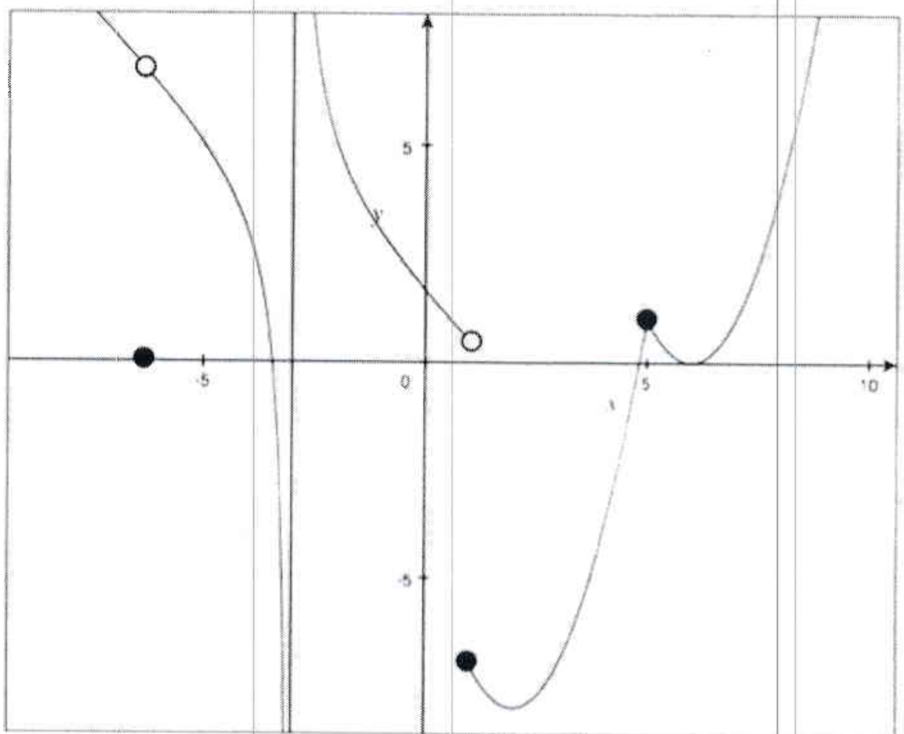
7. What are the three things you have to check to determine the continuity of a function at a point? (6 points)

① the value of $f(x)$ at the point

② the limit of $f(x)$ at the point in question

③ do parts ① & ② agree

8. Discuss the continuity of the function shown in the graph below. State any intervals over which the graph is continuous. (8 points)



$\lim_{x\to -6} f(x) = 6$ graph is discontinuous there since $f(-6) = 0$

$\lim_{x\to -3} f(x) = \text{DNE}$

since $\lim_{x\to -3^-} f(x) = -\infty$

$\lim_{x\to -3^+} f(x) = \infty$

discontinuous there

$\lim_{x\to 1^-} f(x) = \frac{1}{2}$

$\lim_{x\to 1^+} f(x) = -7$

$\lim_{x\to 1} f(x) = \text{DNE}$ discontinuous there

Continuous at all other points in the domain

9. Prove that $\lim_{x\to -3} 4x - 2 = -14$ using the $\epsilon - \delta$ definition of a limit. (12 points)

$|4x-2-(-14)| = |4x+12| = 4|x+3| < \varepsilon$

$|x+3| < \varepsilon/4$ let $\delta = \varepsilon/4$

Suppose $|x-(-3)| = |x+3| < \delta$. and let $\delta = \varepsilon/4$.

then $|x+3| < \varepsilon/4 \Rightarrow 4|x+3| < \varepsilon \Rightarrow |4x+12| < \varepsilon \Rightarrow$

$|4x-2-(-14)| < \varepsilon$. therefore $\lim_{x\to -3} 4x-2 = -14$.

10. Use the limit definition of the derivative to find the equation of the derivative of the following functions (8 points each)

a. $f(x) = x - x^2$

$$\lim_{\Delta x \to 0} \frac{x + \Delta x - (x+\Delta x)^2 - (x - x^2)}{\Delta x} =$$

$$\lim_{\Delta x \to 0} \frac{\cancel{x} + \Delta x - \cancel{x^2} - 2x\Delta x - \Delta x^2 - \cancel{x} + \cancel{x^2}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(1 - 2x - \Delta x)}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 1 - 2x - \Delta x = 1 - 2x$$

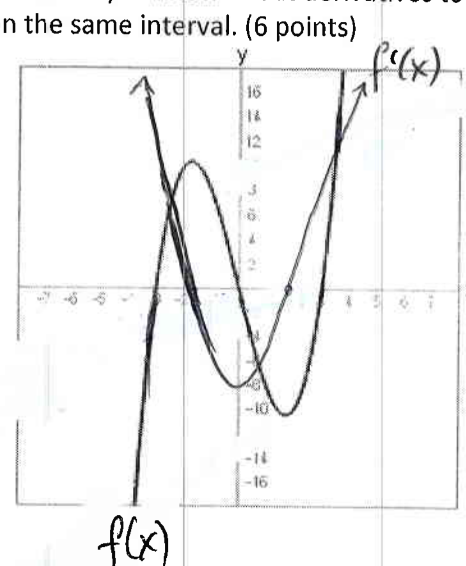
b. $g(x) = \sqrt{2x+1}$

$$\lim_{\Delta x \to 0} \frac{\sqrt{2x+2\Delta x+1} - \sqrt{2x+1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x + 2\Delta x + 1 - (2x+1)}{\Delta x\left(\sqrt{2x+2\Delta x+1} + \sqrt{2x+1}\right)} =$$

$$\lim_{\Delta x \to 0} \frac{\cancel{2x} + 2\cancel{\Delta x} + \cancel{1} - \cancel{2x} - \cancel{1}}{\cancel{\Delta x}\left(\sqrt{2x+2\Delta x+1} + \sqrt{2x+1}\right)} = \lim_{\Delta x \to 0} \frac{2}{\sqrt{2x+2\Delta x+1} + \sqrt{2x+1}} = \frac{2}{2\sqrt{2x+1}}$$

$$= \frac{1}{\sqrt{2x+1}}$$

11. Use what you know about derivatives to sketch a graph of the derivative for the function shown on the same interval. (6 points)



12. Find the first and second derivatives of the following functions. [Only find the first derivative in part d.] (10 points each for a, b, c; 8 points for d)

a. $s(t) = 4\sqrt{t} - \frac{1}{4}t^4 + t + 1$

$4t^{1/2} - \frac{1}{4}t^4 + t + 1$

$s'(t) = 2t^{-1/2} - t^3 + 1 = \frac{2}{\sqrt{t}} - t^3 + 1$

$s''(t) = -t^{-3/2} - 3t^2 = \frac{-1}{\sqrt{t^3}} - 3t^2$

b. $h(x) = (x^2+1)^2 = (x^2+1)(x^2+1) = x^4 + 2x^2 + 1$

$h'(x) = 4x^3 + 4x$

$h''(x) = 12x^2 + 4$

c. $f(x) = x^3 e^x$

$u = x^3 \quad v = e^x$

$u' = 3x^2 \quad v' = e^x$

$f'(x) = 3x^2 e^x + x^3 e^x = e^x(3x^2 + x^3)$

$u = 3x^2 + x^3 \quad v = e^x$

$u' = 6x + 3x^2 \quad v' = e^x$

$f''(x) = (6x + 3x^2)e^x + (3x^2 + x^3)e^x = e^x(x^3 + 6x^2 + 6x)$

d. $q(t) = \frac{2\sqrt{t}-1}{4t+1}$

$q'(t) = \dfrac{\frac{1}{\sqrt{t}}(4t+1) - 4(2\sqrt{t} - 1)}{(4t+1)^2}$

$f = 2t^{1/2} - 1 \quad g = 4t+1$

$f' = t^{-1/2} \quad g' = 4$

13. Find the equation of the tangent line to the graph of $f(x) = x^3 - 4x^2 + 2x - 1$ when $x = 2$. (8 points)

$f'(x) = 3x^2 - 8x + 2$

$f'(2) = 12 - 16 + 2 = -2$

$f(2) = 8 - 16 + 4 - 1 = -5$

$y - y_1 = m(x - x_1)$

$y - (-5) = -2(x - 2)$

$y + 5 = -2x + 4$

$ - 5 \qquad - 5$

$y = -2x - 1$

14. Find any points on the graph of $g(x) = 2x^3 - 3x^2 - 12x + 4$ where the tangent line is horizontal. (8 points)

$g'(x) = 6x^2 - 6x - 12$

$\frac{6x^2 - 6x - 12 = 0}{6}$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2,\ x = -1}$