

3. Integrate by substitution.

a. $\int 2x(x^2 + 4)^8 dx$

$$u = x^2 + 4$$
$$du = 2x dx$$

$$\int u^8 du = \frac{1}{9} u^9 + C = \frac{1}{9} (x^2 + 4)^9 + C$$

b. $\int x e^{x^2} dx$

$$u = x^2$$
$$du = 2x dx \Rightarrow \frac{1}{2} du = x dx$$

$$\frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

c. $\int \frac{x}{(x^2+1)} dx$

$$u = x^2 + 1$$
$$du = 2x dx$$
$$\frac{1}{2} du = x dx$$

$$\frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|x^2 + 1| + C$$

4. Use change of variables to integrate $\int x\sqrt{x-4} dx$.

$$u = \sqrt{x-4}$$
$$u^2 = x-4$$
$$x = u^2 + 4$$
$$dx = 2u du$$

$$\int (u^2 + 4)(u)(2u du) =$$

$$\int 2u^4 + 8u^2 du = \frac{2}{5} u^5 + \frac{8}{3} u^3 + C =$$

$$\frac{2}{5} (x-4)^{5/2} + \frac{8}{3} (x-4)^{3/2} + C$$