

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Find the indicated limits algebraically using properties of limits.

a. $\lim_{x \rightarrow 4} 3x - 7$ $3(4) - 7 = 12 - 7 = 5$

b. $\lim_{x \rightarrow 3} \frac{-5x}{\sqrt{4x-3}}$ $= \frac{-5(3)}{\sqrt{4(3)-3}} = \frac{-15}{\sqrt{12-3}} = \frac{-15}{\sqrt{9}} = \frac{-15}{3} = -5$

c. $\lim_{x \rightarrow 2} \frac{3x^2 - 7x + 2}{2-x}$ $= \lim_{x \rightarrow 2} \frac{(3x-1)(x-2)}{2-x} = -1(3(2)-1) = -(5) = -5$

d. $\lim_{x \rightarrow 0} \frac{\tan(7x)}{\sin(x)}$ $= \lim_{x \rightarrow 0} \frac{\sin(7x)}{\sin(x) \cos(7x)} = \lim_{x \rightarrow 0} \frac{7 \sin(7x)}{7} \cdot \frac{x}{x} \cdot \frac{1}{\cos 7x}$
 $= 7(1)(1)(1) = 7$

e. $\lim_{x \rightarrow 2} \frac{\sin(x-2)}{x^2-4}$ $= \lim_{x \rightarrow 2} \frac{\sin(x-2)}{(x-2)} \cdot \frac{1}{(x+2)} = (1)\left(\frac{1}{4}\right) = \frac{1}{4}$

2. Use the Squeeze Theorem along with a graph, to show that $1 - \frac{x^2}{2} \leq \cos(x) \leq 1$ for x near zero. Find $\lim_{x \rightarrow 0} \cos(x)$.

Since $1 - \frac{x^2}{2} \leq \cos(x) \leq 1$ then

$$\lim_{x \rightarrow 0} 1 - \frac{x^2}{2} \leq \lim_{x \rightarrow 0} \cos(x) \leq \lim_{x \rightarrow 0} 1$$

$$1 \leq \lim_{x \rightarrow 0} \cos(x) \leq 1$$

therefore $\lim_{x \rightarrow 0} \cos(x) = 1$

