Math 1151, Quiz #4, Fall 2014 Name KEY

Instructions: Show all work. Answers without work may only receive partial credit. If you are asked for an explanation, explain as completely as possible. Use exact answers unless specifically asked to round.

1. Prove that $\lim_{x\to 5} 9-2x=-1$ using the $\epsilon-\delta$ definition of a limit.

$|9-2x-(-1)| = |10-2x| = |2x-10| = 2|x-5| < \varepsilon$

$|x-5| < \varepsilon/2 = \delta$

Suppose $|x-5|<\delta$ and let $\delta = \varepsilon/2$ Then

$|x-5| < \varepsilon/2 \Rightarrow 2|x-5| < \varepsilon \Rightarrow |2x-10| = |10-2x| < \varepsilon$

$\Rightarrow |9-2x-(-1)| < \varepsilon$. Thus the $\lim_{x\to 5} 9-2x=-1$

2. Use the limit definition of the derivative to find the derivative of $f(x)=x^2-5$ and evaluate the derivative at the point $P(3,4)$.

$f'(x) = \lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-5-(x^2-5)}{\Delta x} =$

$\lim_{\Delta x\to 0} \frac{\cancel{x^2}+2x\Delta x+\Delta x^2-\cancel{5}-\cancel{x^2}+\cancel{5}}{\Delta x} = \lim_{\Delta x\to 0} \frac{\cancel{\Delta x}(2x+\Delta x)}{\cancel{\Delta x}} =$

$\lim_{\Delta x\to 0} 2x+\Delta x = 2x$

at the point $(3,4)$ $x=3$ so $f'(3) = 2(3) = 6$