Math 1151, Simpson's Rule Activity, Fall 2014 Name KEY

Instructions: Use the numerical integration technique called Simpson's Rule (the formula is given below) to approximate the area under the under the curve on the given interval with the specified value of n.

$f(x) \approx \frac{b-a}{3n}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)]$, for n **even**.

1. $\int_0^2 x^3\, dx, n = 4$

$\Delta x = \frac{2}{4} = \frac{1}{2}$

$x_0 = 0,\ x_1 = \frac{1}{2}\ x_2 = 1\ x_3 = \frac{3}{2}\ x_4 = 2$

$$\frac{2-0}{12}\left[0 + 4\left(\tfrac{1}{8}\right) + 2(1) + 4\left(\tfrac{27}{8}\right) + 8\right] =$$

$$\frac{1}{6}\left[\tfrac{1}{2} + 2 + \tfrac{27}{2} + 8\right] = \frac{1}{6}[10 + 14] = \frac{24}{6} = 4$$

2. $\int_1^4 \ln(x)\, dx, n = 6$

$\Delta x = \frac{3}{6} = \frac{1}{2}$ $x_0 = 1,\ x_1 = \frac{3}{2},\ x_2 = 2,$
$x_3 = \frac{5}{2},\ x_4 = 3,\ x_5 = \frac{7}{2},\ x_6 = 4$

$$\frac{3}{18}\left[\ln(1) + 4\left(\ln\tfrac{3}{2}\right) + 2\ln 2 + 4\ln\tfrac{5}{2} + 2\ln 3 + 4\ln\tfrac{7}{2} + \ln 4\right]$$

$$\frac{1}{6}[15.26788853] \approx 2.544648\ldots$$

3. $\int_0^{\frac{\pi}{4}} x\tan(x)\, dx, n = 8$

$\Delta x = \frac{\frac{\pi}{4}}{8} = \frac{\pi}{32}$

$x_0 = 0,\ x_1 = \frac{\pi}{32},\ x_2 = \frac{\pi}{16},\ x_3 = \frac{3\pi}{32},$
$x_4 = \frac{\pi}{8},\ x_5 = \frac{5\pi}{32},\ x_6 = \frac{3\pi}{16},\ x_7 = \frac{7\pi}{32},$
$x_8 = \frac{\pi}{4}$

$$\frac{\frac{\pi}{4}}{24}\left[\tan 0 + 4\tfrac{\pi}{32}\tan\tfrac{\pi}{32} + 2\cdot\tfrac{\pi}{16}\tan\tfrac{\pi}{16} + 4\tfrac{3\pi}{32}\tan\tfrac{3\pi}{32} + 2\cdot\tfrac{\pi}{8}\tan\tfrac{\pi}{8} + 4\cdot\tfrac{5\pi}{32}\tan\tfrac{5\pi}{32} + 2\cdot\tfrac{3\pi}{16}\tan\tfrac{3\pi}{16} + 4\cdot\tfrac{7\pi}{32}\tan\tfrac{7\pi}{32} + \tfrac{\pi}{4}\tan\tfrac{\pi}{4}\right] =$$

$$\frac{\pi}{96}[5.677531041] = .18579676\,88$$

Instructions: Because the Trapezoidal Rule and Simpson's Rule are approximations, there is an error calculation associated with each method. The error on the Trapezoidal Rule is $|E| \le \frac{(b-a)^3}{12n^2}[\max|f''(x)|]$, and the error on Simpson's Rule is $|E| \le \frac{(b-a)^5}{180n^4}[\max|f^{IV}(x)|]$.

4. For each of the problems above, calculate the error using these formula for approximating the integral for the specified value for n for both the Trapezoidal Rule and Simpson's Rule.

5. For each of the problems above, use the same error formulas to calculate the value of n needed for each approximation method to get the error under $|E| \le 0.001$ of the true value. Remember to round n for the Trapezoidal Rule to the next larger integer, and for Simpson's Rule to the next larger even integer.

1. $\frac{(2-0)^3}{12n^2}(12) = \frac{8}{\cancel{12}(4)^2}(\cancel{12}) = \frac{8}{16} = \frac{1}{2}$ trap.

$f' = 3x^2$ $f'' = 6x$ $f''' = 6$ $f^{IV} = 0$

$E = 0$ Simp

2. $\frac{\left(4-1\right)^3}{12n^2}(1) = \frac{27}{12(6)^2} = \frac{\cancel{27}}{4\cdot\cancel{3}\cdot 4\cdot\cancel{9}} = \frac{1}{16}$ trap

$f' = \frac{1}{x}$ $f'' = -x^{-2}$ $f''' = 2x^{-3}$ $f^{IV} = -6x^{-4}$

$\frac{243}{180(6)^4}(6) = 1/160$ Simp.

3. $\frac{\left(\frac{\pi}{4}\right)^3}{12(8)^2}\left(2(\sqrt{2})^2(1)\right) = \frac{\pi^3}{64\cdot 12\cdot \cancel{64}_{16}}\cancel{4}$

$= \frac{\pi^3}{12288}$ Trap.

$\frac{\pi^5}{180(8)^4\cdot 4^5}\cdot 80 = \frac{\pi^5}{1943784}$ Simp.

$f' = \sec^2 x$ $f'' = 2\sec^2 x\tan x$

$f''' = 4\sec^2 x\tan^2 x + 2\sec^4 x$

$f^{IV} = 8\sec^2 x\tan^3 x + 8\sec^4 x + 8\sec^4 x\tan x$

$8(\sqrt{2})^2(1)^3 + 8(\sqrt{2})^4 + 8(\sqrt{2})^4(1) =$

$8(2) + 8(4) + 8(4) = 16 + 64 = 80$

1. Trap. $\frac{8}{\cancel{12}n^2}(\cancel{12}) \le .001$

$n^2 \ge 8000$ $n \ge 90$

Simp $0 \Rightarrow n = 2$

2. $\frac{27}{12n^2} \le .001 \Rightarrow \frac{1000\cdot 12}{27} \le n^2$

$n \ge 21.08$

$n \ge 22$

$\frac{243}{180n^4} \le .001$ $\frac{1000\cdot 180}{243} \le n^4$

$n \ge 5.266$

$n = 6$

3. $\frac{\pi^3}{4^3\cdot 12n^2}(4) \le .001$

$\frac{1000}{\pi^3\cdot 4}(12\cdot 4^3) \le n^2$ $n \ge 79$

$\frac{\pi^5}{180n^4(4)^5}\cdot 80 \le .001$

$\frac{1000}{80\pi^5}\cdot 180\cdot 4^5 \le n^4$

$n \ge 9.315$

$n \ge 10$