Math 1151, U-Substitution & Change of Variables Activity, Fall 2014 Name KEY

Instructions: For each of the functions below, use u-substitution or change of variables as appropriate to integrate the function.

1. $\int e^{\tan(2x)} \sec^2 2x\, dx$

$u = \tan 2x$

$du = 2\sec^2 2x\, dx \Rightarrow \frac{1}{2} du = \sec^2 2x\, dx$

$\int e^u \cdot \frac{1}{2} du = \frac{1}{2}\int e^u du = \frac{1}{2} e^{\tan 2x} + C$

2. $\int 3^{x/2} dx$

$u = x/2 = \frac{1}{2}x$

$du = \frac{1}{2} dx \Rightarrow 2du = dx$

$\int 3^u \cdot 2du = 2\int 3^u du = \frac{2}{\ln 3} \cdot 3^{x/2} + C$

3. $\int x^2\sqrt{x^3+2}dx$

$u = x^3 + 2$

$du = 3x^2 dx \Rightarrow \frac{1}{3} du = x^2 dx$

$\int u^{1/2} \cdot \frac{1}{3} du = \frac{1}{3}\int u^{1/2} du = \frac{1}{3} \cdot \frac{2}{3} u^{3/2} \Rightarrow \frac{2}{9}(x^3+2)^{3/2} + C$

4. $\int \frac{1}{x^2} e^{3/x}\, dx$

$u = \frac{3}{x} = 3x^{-1}$

$du = -3x^{-2}dx \Rightarrow -\frac{1}{3}du = \frac{1}{x^2}dx$

$\int -\frac{1}{3} e^u du = -\frac{1}{3}\int e^u du = -\frac{1}{3} e^{3/x} + C$

5. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

$u = 1 + \sqrt{x} = 1 + x^{1/2}$

$du = \frac{1}{2} \cdot x^{-1/2} dx \Rightarrow 2du = \frac{1}{\sqrt{x}} dx$

$\int \frac{1}{(1+\sqrt{x})^2} \cdot \frac{1}{\sqrt{x}} dx = \int \frac{1}{u^2} \cdot 2du = 2\int u^{-2} du = 2\frac{u^{-1}}{-1} + C$

$= -2(1+\sqrt{x})^{-1} + C = \frac{-2}{1+\sqrt{x}} + C$

6. $\int x\sqrt{4x+1}dx$

$u=\sqrt{4x+1} \qquad u^2=4x+1 \Rightarrow \frac{u^2-1}{4}=x$

$dx = \frac{2u}{4}du = \frac{1}{2}u\,du$

$\int \frac{u^2-1}{4}\cdot u \cdot \frac{1}{2}u\,du = \frac{1}{8}\int (u^2-1)u^2\,du = \frac{1}{8}\int u^4-u^2\,du =$

$\frac{1}{8}\left[\frac{u^5}{5}-\frac{u^3}{3}\right]+C = \frac{1}{40}(4x+1)^{5/2} - \frac{1}{24}(4x+1)^{3/2}+C$

7. $\int (x+1)\sqrt{2-x}dx$

$u=\sqrt{2-x} \Rightarrow u^2=2-x \Rightarrow 2-u^2=x$

$-2u\,du=dx \qquad x+1=3-u^2$

$\int (3-u^2)u(-2u\,du) = -2\int(3-u^2)u^2\,du = -2\int 3u^2-u^4\,du$

$=-2\left[u^3-\frac{1}{5}u^5\right]+C = -2(2-x)^{3/2}+\frac{2}{5}(2-x)^{5/2}+C$

8. $\int \frac{2x+1}{\sqrt{x+4}}dx$

$u=\sqrt{x+4} \qquad u^2-4=x \qquad 2u\,du=dx$

$2(u^2-4)+1 = 2u^2-7 = 2x+1$

$\int \frac{(2u^2-7)\cdot 2\cancel{u}\,du}{\cancel{u}} = 2\int 2u^2-7\,du = 2\left[\frac{2}{3}u^3-7u\right]+C$

$=\frac{4}{3}(x+4)^{3/2}-14\sqrt{x+4}+C$