Stat 2470, Exam #1, Fall 2014 Name KEY

Instructions: Show all work. Use exact answers or appropriate rounding conventions. If you use your calculator, you can show work by saying which calculator commands you used.

1. a. Determine the mean and standard deviation for the rainfall amount from seeded clouds shown below. (3 points)

4.1	7.7	17.5	31.4	32.7	40.6	92.4	115.3	118.3	119.0
129.6	198.6	200.7	242.5	255.0	274.7	302.8	334.1	430.0	489.1
703.4	978.0	1656.0	1697.8	2745.6					

1 Var Stats: $\bar{x} = 448.676$

$s = 663.29$

b. Find the median, and the 10% trimmed mean. (4 points)

$\boxed{\tilde{x} = 200.7}$

$n = 25.$

10% × 2.5 values

$\bar{x}_{tr(2)\text{ values}} : 321.9857$

$\boxed{\bar{x}_{tr(.10)} = 294.89}$

$\bar{x}_{tr(3)\text{ values}} = 267.8$

c. Construct a boxplot for the same data. Label your primary axis. (3 points)

min = 4.1

$Q_1 = 66.5$

$Q_2 = \text{Med} = 200.7$

$Q_3 = 459.55$

Max = 2745.6

or

2. A college library has five copies of a certain text on reserve. Two copies (we'll number them 1 and 2) are first printings, and the other three (3, 4, 5) are second printings. A student examines the books in random order, stopping when they select a second printing. What are all the events in the sample space? (5 points)

{3} {2,5}
{4} {1,2,3}
{5} {2,1,3}
{1, 3} {1,2,4}
{1,4} {2,1,4}
{1,5} {1,2,5}
{2,3} {2,1,5}
{2,4}

3. What are the three conditions on probability? (3 points)

$p \geq 0$

$p \leq 1$

$\sum p = 1$

4. An individual is presented with three different glasses of cola, labeled C, D and P. He is asked to taste all three and then list them in order of preference. Suppose the same cola has actually been put in all three glasses.
   a. What are all the simple events in this ranking experiment? What are the probabilities you assign to each. (3 points)

   {C, D, P} {P, D, C}
   {C, P, D} {D, P, C}
   {P, C, D} {D, C, P}

   all equal = 1/6 each ep

   b. What is the probability that C is ranked first? (3 points)

   $\frac{2}{6} = \frac{1}{3}$

   c. What is the probability that C is ranked first and D is ranked last? (2 points)

   1/6

5. Suppose that a math department has 35 full-time faculty members. They need to choose four people to serve on 4 different committee. How many different ways can this be done? (4 points)

$35P4 = 1,256,40$

6. Suppose that the same math department needs to choose six people for a single committee. How many different ways can this be done? (4 points)

$35C6 = 1,623,160$

7. Suppose that a college has four divisions: business with 56 faculty, arts & sciences with 145 faculty, allied health with 45 faculty, and career and tech with 78 faculty. How many ways are there to choose a committee of 4 with one faculty member from each division? (4 points)

$56 \cdot 145 \cdot 45 \cdot 78 = 28{,}501{,}200$

8. Suppose that a math department with 35 faculty members is divided into 20 men and 15 women. What is the probability of forming a committee of six with exactly three men and three women if all members are chosen at random? (5 points)

$\binom{20}{3}\binom{15}{3} = 518{,}700$

9. A department store sells shirts in three sizes and in three patterns (small, medium and large; plaid, print and stripes). The table below gives the number of shirts of each type sold on a particular, typical day.

Size	**Plaid**		**Print**		**Stripes**		
Small	3	.068	2	.045	3	.068	8
Medium	10	.227	5	.1136	7	.159	22
Large	4	.091	2	.045	8	.182	14
	17		9		18		144

a. What is the probability that the next shirt sold is a medium print shirt? (3 points)

$\frac{5}{44} = .1136$

b. What is the probability that the next shirt sold will be a plaid shirt? (3 points)

$\frac{17}{44} = .386$

c. Given that the next shirt is plaid, what is the probability that it is a small size? (3 points)

$\frac{3}{17} = .176$

d. Given that the next shirt is small size, what is the probability that the shirt is plaid? (3 points)

$\frac{3}{8} = .375$

10. Rolling a fair 12-sided-die and flipping a fair coin are independent events. Calculate the following probabilities.

a. What is the probability of getting a 7 or 11 on the die? (2 points)

$$\frac{2}{12} = \frac{1}{6} \approx .17$$

b. What is the probability of getting a 7 or 11, and a head? (3 points)

$$\frac{2}{12} \cdot \frac{1}{5} = \frac{1}{30} = .033$$

c. What is the probability of getting a 7 or 11, or a head? (3 points)

$$\frac{2}{12} + \frac{1}{5} - \frac{1}{30} = \frac{1}{3} = .33$$

11. Consider the following probability table for a particular course's grade distributions where A=4, B=3, C=2, D=1, E/F=0.

x	4	3	2	1	0
$p(x)$	0.2	0.35	0.25	0.15	0.05

a. Find the expected value of the probability distribution. (4 points)

$$E(X) = 4(.2) + 3(.35) + 2(.25) + 1(.15) + 0(.05) = 2.5$$

b. Calculate the variance of the distribution. (4 points)

$$E(X^2) = 16(.2) + 9(.35) + 4(.25) + 1(.15) + 0(.05) = 7.5$$

$$V(X) = E(X^2) - E(X) = 7.5 - (2.5)^2 = 1.25$$

c. What is the probability that a randomly selected student who took the course received a B or higher? (3 points)

$$.2 + .35 = .55$$

d. Suppose that 15 students take the course next semester. What is the probability that no more than three students will receive a D or less? (3 points)

$p(\text{D or less}) = .2$

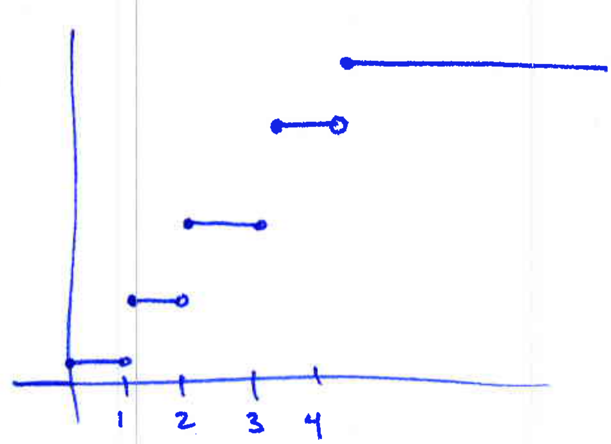
$\text{binomialcdf}(15, .2, 3) = .648$

e. Given that a particular student passed the class, what is the probability that they received an A? (3 points)

$$\frac{.2}{.2+.35+.25} = \frac{.2}{.8} = .25$$

f. Give the cumulative distribution function F(x) in correct piecewise notation. Sketch the graph. (3 points)

$$F(x) = \begin{cases} 0 & x < 0 \\ .05 & 0 \le x < 1 \\ .2 & 1 \le x < 2 \\ .45 & 2 \le x < 3 \\ .8 & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$



12. Suppose that 90% of all batteries from a certain factory have acceptable voltages. A certain type of flashlight requires two D batteries, and the flashlight will work only if both batteries have acceptable voltages. Among ten randomly selected flashlights, what is the probability that that at least nine will work? (6 points)

$.9 * .9 = .81$ both work

$\binom{10}{9}(.81)^9(.19)^1 + \binom{10}{10}(.81)^{10}(.19)^0$ or

$1 - \text{binomialcdf}(10, .81, 8) = .4067$

13. The number of requests for assistance received by a towing service is a Poisson process with a rate of four per hour.

a. Compute the probability that exactly ten requests are received during a particular two-hour period. (4 points)

$\mu = 4/hr$

$\mu = 8/hr$

poissonpdf(8,10) = .09926

or $\frac{e^{-8} 8^{10}}{10!}$

b. If the operators of the towing service take a 30-min break for lunch, with is the probability that they don't miss any calls for assistance? (4 points)

$\mu = 2/(1/2\, hr)$

poissonpdf(2,0) = .135

or $\frac{e^{-2} 2^0}{0!}$

14. Consider the probability density function $f(x) = \begin{cases} \frac{k}{x^4}, & x > 1 \\ 0, & x \le 1 \end{cases}$.

a. Determine the value of k for which this is a legitimate probability density function. (6 points)

$k \int_1^\infty x^{-4} dx = k\left[\frac{x^{-3}}{-4}\right|_1^\infty = k\left[-\frac{1}{4}\frac{1}{x^3}\right|_1^\infty =$

$-\frac{k}{4}[0-1] = \frac{k}{4} = 1 \Rightarrow \underline{k=4}$

b. Find $P(1 < X < 2)$. (4 points)

$\int_1^2 \frac{4}{x^4} dx = -\frac{1}{x^3}\Big|_1^2 = -\frac{1}{8} + \frac{1}{1} = \frac{7}{8} = .875$

c. What is the expected value of X. (5 points)

$E(x) = \int_1^\infty \frac{4}{x^3} dx = \frac{-4}{2x^2}\Big|_1^\infty = -2(0) + 2(1) = 2$

d. What is the variance of X? (5 points)

$V(x) = E(x^2) - [E(x)]^2 = \int_1^\infty \frac{4}{x^2} dx - 2^2 = \frac{-4}{x}\Big|_1^\infty - 4 =$

$-(0) + 4 - 4 = 0$

15. Find the 92nd percentile and the 37th percentile for the standard normal distribution. (6 points)

invNorm(.92) = 1.405

invNorm(.37) = −.3318...

16. Mopeds have a maximum speed that is normally distributed with a mean of 46.8 kph and standard deviation of 1.75 kph.

a. What is the probability that a randomly selected moped has a maximum speed of at least 50 kph? (6 points)

1 − normalcdf(−E99, 50, 46.8, 1.75) =

normalcdf(50, E99, 46.8, 1.75) = .0337

b. What is the probability that maximum speed differs from the mean by more than 1.5 standard deviations? (6 points)

normalcdf(−1.5, 1.5) = .866

or normalcdf(44.175, 49.425, 46.8, 1.75)

1.5 × 1.75 = 2.625 46.8 − 2.625 = 44.175, 46.8 + 2.625 = 49.425

17. Suppose that X has a standard gamma distribution (with $\beta = 1$), with $\alpha = 5$, find $P(X < 5)$ and $P(2 < X < 6)$. (8 points)

$f_\Gamma(x;\alpha) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \quad x \geq 0$

$f(x;5) = \frac{1}{4!} x^4 e^{-x}$

$P(X<5) = \int_0^5 \frac{1}{24} x^4 e^{-x}\,dx = .5595$

$P(2<X<6) = \int_2^6 \frac{1}{24} x^4 e^{-x}\,dx = .66229$