Stat 2470, Quiz #6, Fall 2014 Name KEY

Instructions: Show all work. Use exact answers or appropriate rounding conventions. If you use your calculator, you can show work by saying which calculator commands you used.

1. Rainfall duration is modeled exponentially with a mean of 2.725 hours. What is the probability that the duration of a particular storm will last more than 2 hours? At least 3 hours?

$\mu = \frac{1}{\lambda} = 2.725 \Rightarrow \lambda = \frac{40}{109}$

$\int_2^\infty \frac{40}{109} e^{-\frac{40}{109}x} dx = -e^{-\frac{40}{109}x}\Big|_2^\infty = .48\ldots$

$1 - \int_0^3 \frac{40}{109} e^{-\frac{40}{109}x} dx = -e^{-\frac{40}{109}}\Big|_0^3 = \left(1 - e^{-\frac{40}{109}(3)}\right) = .3325\ldots$

2. Find the values of $\Gamma(7)$ and $\Gamma\left(\frac{7}{2}\right)$.

$\Gamma(7) = 6! = 720$

$\Gamma(7/2) = 5/2 \cdot 3/2 \cdot 1/2\, \Gamma(1/2) =$

$5/2 \cdot 3/2 \cdot 1/2 \sqrt{\pi} \approx 3.32$

$(5/2)!$ in calculator

3. Suppose the time spent by a randomly selected student who uses a terminal connected to a local time-sharing computer has a gamma distribution with a mean of 20 and a variance of 80. If $E(X) = \alpha\beta, V(X) = \alpha\beta^2$, find the gamma distribution that models the data.

$\alpha\beta = 20$

$\alpha\beta^2 = 80 = (\alpha\beta)\beta = 20\beta = 80$

$\beta = 4 \quad \alpha = 5$

$f_{gamma} = \begin{cases} \frac{1}{4^5 \Gamma(5)} x^4 e^{-x/4} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

or $\begin{cases} \frac{1}{24576} x^4 e^{-x/4} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$