Stat 2470, Quiz #8, Fall 2014 Name KEY

Instructions: Show all work. Use exact answers or appropriate rounding conventions. If you use your calculator, you can show work by saying which calculator commands you used.

1. Find $E(X), E(Y), E(XY)$ for the joint probability distribution $f(x) = kx^3y,\ 0 \le x \le 4, 0 \le y \le 8-2x$.

$$\int_0^4 \int_0^{8-2x} kx^3y \, dy\,dx = \int_0^4 \frac{kx^3y^2}{2}\Big|_0^{8-2x} dx = \int_0^4 \frac{kx^3}{2}(8-2x)^2 dx =$$

$$k \cdot \frac{2048}{15} = 1 \Rightarrow k = \frac{15}{2048}$$

$$E(X) = \int_0^4 \int_0^{8-2x} \frac{15}{2048} x^4 y \, dy\,dx = \int_0^4 \frac{15}{4096} x^4 (8-2x)^2 dx = \frac{16}{7}$$

$$E(Y) = \int_0^4 \int_0^{8-2x} \frac{15}{2048} x^3 y^2 \, dy\,dx = \int_0^4 \frac{5}{2048} x^3 (8-2x)^3 dx = \frac{16}{7}$$

$$E(XY) = \int_0^4 \int_0^{8-2x} \frac{15}{2048} x^4 y^2 \, dy\,dx = \int_0^4 \frac{5}{2048} x^4 (8-2x)^3 dx = \frac{32}{7}$$

2. In your own words, describe the Central Limit Theorem and the importance of its role in statistics.

The central limit Theorem allows us to approximate sampling distributions as normal centered around the true value of a statistic even when the underlying distribution is not normal. And our approximations get better as sample sizes increase.