Math 2568, Quiz #11, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -6 \\ 2 & -6 \end{bmatrix}$ and use that information to find a similarity transformation that diagonalizes the matrix.

$(1-\lambda)(-6-\lambda)+12=0$

$-6-\lambda+6\lambda+\lambda^2+12=0$

$\lambda^2+\lambda+6=0$

$P=\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$(\lambda+2)(\lambda+3)=0$

$\lambda=-2,-3$

$\begin{bmatrix} 3 & -6 \\ 2 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$

$x_1-2x_2=0$

$x_1=2x_2$

$x_2=x_2$ $\vec{v}_1=\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 4 & -6 \\ 2 & -3 \end{bmatrix} \to \begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix}$ $\cancel{2}x_1=\frac{3x_2}{2}$

$x_2=x_2$

$\vec{v}_2=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

2. Use the similarity transformation found above to find the value of e^A.

$\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} e^{-2} & 0 \\ 0 & e^{-3} \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} e^{-2} & 0 \\ 0 & e^{-3} \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}=$

$\begin{bmatrix} 2e^{-2} & -3e^{-3} \\ -2e^{-2} & 2e^{-3} \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 4e^{-2}-3e^{-3} & 6e^{-2}-6e^{-3} \\ -2e^{-2}+2e^{-3} & -3e^{-2}+4e^{-3} \end{bmatrix}$

3. Find the eigenvalues and eigenvectors of the matrix $B = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$.

$(1-\lambda)(3-\lambda)+2=0$

$3-\lambda-3\lambda+\lambda^2+2=0$

$\lambda^2-4\lambda+5=0$

$\begin{bmatrix} 1-(2+i) & -2 \\ 1 & 3-(2+i) \end{bmatrix}=\begin{bmatrix} -1-i & -2 \\ 1 & 1-i \end{bmatrix}$

$x_1=(-1+i)x_2$

$x_2=x_2$

$\lambda=\frac{4\pm\sqrt{16-20}}{2}=\frac{4\pm 2i}{2}=2\pm i$

$\vec{V}=\begin{bmatrix} -1 \\ 1 \end{bmatrix}+\begin{bmatrix} 1 \\ 0 \end{bmatrix}i$