Math 2568, Quiz #15, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Given the vectors $\left\{\begin{bmatrix}2\\-5\\1\end{bmatrix}, \begin{bmatrix}4\\-1\\2\end{bmatrix}\right\}$, use the Gram-Schmidt process to create an orthonormal basis for the subspace.

$\vec{v}_1 = \begin{bmatrix}2\\-5\\1\end{bmatrix}$

$\vec{v}_2 = \begin{bmatrix}4\\-1\\2\end{bmatrix} - \frac{8+5+2}{4+25+1}\begin{bmatrix}2\\-5\\1\end{bmatrix} = \begin{bmatrix}4\\-1\\2\end{bmatrix} - \frac{15}{30}\begin{bmatrix}2\\-5\\1\end{bmatrix} = \begin{bmatrix}4\\-1\\2\end{bmatrix} + \begin{bmatrix}-1\\5/2\\-1/2\end{bmatrix}$

$= \begin{bmatrix}3\\3/2\\3/2\end{bmatrix} \to \begin{bmatrix}6\\3\\3\end{bmatrix} \to \begin{bmatrix}2\\1\\1\end{bmatrix}$

$B = \left\{\begin{bmatrix}2/\sqrt{30}\\-5/\sqrt{30}\\1/\sqrt{30}\end{bmatrix}, \begin{bmatrix}2/\sqrt{6}\\1/\sqrt{6}\\1/\sqrt{6}\end{bmatrix}\right\}$

2. Use the information obtained above to find the QR factorization of the matrix $A = \begin{bmatrix}2 & 4\\-5 & -1\\1 & 2\end{bmatrix}$.

$Q = \begin{bmatrix}2/\sqrt{30} & 2/\sqrt{6}\\-5/\sqrt{30} & 1/\sqrt{6}\\1/\sqrt{30} & 1/\sqrt{6}\end{bmatrix}$

$R = Q^TA = \begin{bmatrix}2/\sqrt{30} & -5/\sqrt{30} & 1/\sqrt{30}\\2/\sqrt{6} & 1/\sqrt{6} & 1/\sqrt{6}\end{bmatrix}\begin{bmatrix}2 & 4\\-5 & -1\\1 & 2\end{bmatrix} = \begin{bmatrix}4/\sqrt{30}+25/\sqrt{30}+1/\sqrt{30} & 8/\sqrt{30}+5/\sqrt{30}+2/\sqrt{30}\\4/\sqrt{6}-5/\sqrt{6}+1/\sqrt{6} & 8/\sqrt{6}-1/\sqrt{6}+2/\sqrt{6}\end{bmatrix}$

$= \begin{bmatrix}\sqrt{30} & \frac{\sqrt{30}}{2}\\0 & \frac{3\sqrt{6}}{2}\end{bmatrix}$