Math 2568, Quiz #16, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Find a least squares solution to the of $A\vec{x} = \vec{b}$ for $A = \begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix}, \vec{b} = \begin{bmatrix} 3 \\ 1 \\ -4 \\ 2 \end{bmatrix}$.

$(A^TA)^{-1}A^T\vec{b} = \vec{x}$

$\begin{bmatrix} 1 & -1 & 0 & 2 \\ -2 & 2 & 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 2 \\ 0 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & 42 \end{bmatrix}$

$(A^TA)^{-1} = \begin{bmatrix} 7/36 & -1/36 \\ -1/36 & 1/36 \end{bmatrix}$

$(A^TA)^{-1}A^T\vec{b} = \begin{bmatrix} 4/3 \\ -1/3 \end{bmatrix} = \vec{x}$

2. Find a least squares regression model for a quadratic $y = \beta_0 + \beta_1 x + \beta_2 x^2$ for the points $(1,0.5), (2,1.2), (3,3.5), (4, 5.6), (5, 9.8), (5,13.1), (6,22.7)$.

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \\ 1 & 5 & 25 \\ 1 & 6 & 36 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} .5 \\ 1.2 \\ 3.5 \\ 5.6 \\ 9.8 \\ 13.1 \\ 22.7 \end{bmatrix}$

$\beta_0 + \beta_1(1) + \beta_2(1)^2 = .5$
$\beta_0 + \beta_1(2) + \beta_2(2)^2 = 1.2$
$\beta_0 + \beta_1(3) + \beta_2(3)^2 = 3.5$
$\beta_0 + \beta_1(4) + \beta_2(4)^2 = 5.6$
$\beta_0 + \beta_1(5) + \beta_2(5)^2 = 9.8$
$\beta_0 + \beta_1(5) + \beta_2(5)^2 = 13.1$
$\beta_0 + \beta_1(6) + \beta_2(6)^2 = 22.7$

$\vec{x} = (A^TA)^{-1}A^T\vec{b} = \begin{bmatrix} 2799/610 \\ -4.48598 \\ 1.21479 \end{bmatrix} \approx \begin{bmatrix} 4.588 \\ -4.486 \\ 1.215 \end{bmatrix}$

$y = 4.588 - 4.486x + 1.215x^2$