Math 2568, Quiz #4, Fall 2014

Name



**Instructions**: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

- 1. Find the linear transformation with the following properties.
  - a. The 2x2 rotation matrix that rotates a vector by the angle  $\frac{2\pi}{3}$ .

$$\begin{bmatrix} -Y_2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -\frac{1}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3}/2 & -\frac{1}{2} \end{bmatrix}$$

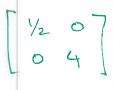
b. The 2x2 projection matrix that maps a vector onto the  $x_2$  axis.



c. The 2x2 sheer matrix that maps the vector  $\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} k \\ 1 \end{bmatrix}$ .



d. The 2x2 matrix that compresses  $x_1$  by a factor of  $\frac{1}{2}$  and stretches  $x_2$  by a factor of 4.



2. Find the null space of the matrix below. Write the solution in the form  $span\{v_1, v_2, ..., v_k\}$ .

$$A = \begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & -6 & 5 & 0 \\ 0 & 0 & 0 & 1 & 4 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & 0 & 29 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 2x_2 & -3x_3 & -29x_3 \\ x_1 = & x_2 \\ x_2 = & x_3 \\ x_4 = & -4x_5 \\ x_5 = & x_5 \\ x_5 = & x_5 \\ x_6 = & 0 \end{bmatrix}$$