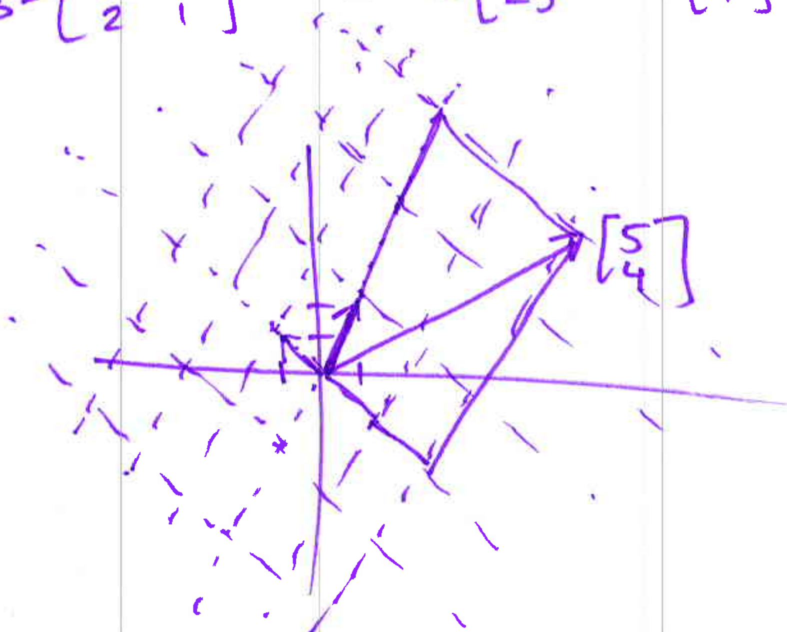
Math 2568, Quiz #8, Fall 2014 Name KEY

Instructions: Show all work. Answer each question as completely as possible. Use exact values (yes, that means fractions!).

1. Graphically interpret the meaning of the following information: A basis for R^2 is $\left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}-1\\1\end{bmatrix}\right\}$; the coordinates for $[\vec{x}]_B = \begin{bmatrix}3\\-2\end{bmatrix}$. Sketch the situation and find the vector $\vec{x}$ in the standard basis.

$P_B = \begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}$ $\vec{x} = 3\begin{bmatrix}1\\2\end{bmatrix} + (-2)\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}5\\4\end{bmatrix}$



2. Determine if the polynomials $\{(2-t)^3, (3-t)^2, 1+6t-5t^2+t^3, 1\}$ form a basis for the space P_3. Explain your reasoning.

$(2-t)^3 = 8-12t+6t^2-t^3$ $(3-t)^2 = 9-6t+t^2$

$\begin{bmatrix}8 & 9 & 1 & 1\\-12 & -6 & 6 & 0\\6 & 1 & -5 & 0\\-1 & 0 & 1 & 0\end{bmatrix} \Rightarrow \text{rref} \Rightarrow \begin{bmatrix}1 & 0 & -1 & 0\\0 & 1 & 1 & 0\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{bmatrix}$

not a basis for P_3 since there are only 3 pivots but 4 vectors needed

3. Use the set of polynomials in #2. Find a basis for the subspace, if needed, and represent the polynomial $4+t-t^2+2t^3$ in the given basis. If the vector is not in the subspace, explain why not. Clearly state any change-of-basis matrices you employ.

Basis Supsp. $\left\{\begin{bmatrix}8\\-12\\6\\-1\end{bmatrix},\begin{bmatrix}9\\-6\\1\\0\end{bmatrix},\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\right\}$

$\left[\begin{array}{ccc|c}8 & 9 & 1 & 4\\-12 & -6 & 0 & 1\\6 & 1 & 0 & -1\\-1 & 0 & 0 & 2\end{array}\right] \Rightarrow \text{rref} \left[\begin{array}{ccc|c}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{array}\right] \leftarrow$ inconsistent

this polynomial is not in the subspace