# Algebraic Equations & Algebraic Expressions

## Vocabulary:

An **algebraic expression** is an algebraic statement (involving numbers and variables) without an equal sign. Algebraic expressions can be simplified, but cannot be "solved" since the expression isn't equal to anything standing by itself.

An **algebraic equation** is an algebraic statement with an equal sign: two expressions are set equal to each other, or in other words two expressions are said to be *equivalent* (have the same solution). Algebraic equations can be both simplified and solved for a variable.

A **linear equation** in <u>one</u> variable is defined as $\{ax+b=c \mid a,b,c \in R, a \neq 0\}$.

In English, this says that in the linear equation *ax+b=c*, all the values of a, b and c are real numbers, and *a* cannot be equal to zero (so that the variable does not vanish). Linear equations have no powers of the variable, nor does it do anything more exotic like square rooting or dividing the variable.[1]

A **solution** of an equation is a replacement for the variable that makes the statement true.

The **solution set** is a set of all possible solutions to an equation (or inequality).

**To solve an equation** write a series of *equivalent equations* until x is equivalent to some number.

## Properties of Equality:

In addition to general simplifying which we can do with expressions (such as the *associative* and *commutative* properties, the *distributive* property, *combining like terms*, *finding a common denominator*, etc.) there are additional operations we can perform on equations that cannot be performed on expressions owing to the equal sign. Students often find it tempting to simply insert an =0 into an expression so that they can do these steps as well, but don't do it! When an expression says **simplify** only the properties listed in the parentheses above can be employed. When an equation says **solve**, only then can the following properties be used.

### Addition Property of Equality:

Given real numbers a, b, c: If *a=b*, then *a+c=b+c.*
You can add or subtract the same number from both sides of an equation and the result is an expression equivalent to the original.

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[1] We will see why these expressions are called *linear* when we do algebraic expressions in two variables, however, for now, remember that these are the simplest kind of algebraic equation, and we generally will want to convert all more complicated equations into this form if at all possible. The techniques for doing this are much of what algebra is about.

Example.
Solve for x.

① $28 - x = 2x + 7$

② $$\begin{array}{r} 28 - x = 2x + 7 \\ \underline{\quad +x \quad +x} \\ 28 \quad = 3x + 7 \end{array}$$

③ $$\begin{array}{r} 28 = 3x + 7 \\ \underline{-7 \qquad -7} \\ 21 = 3x \end{array}$$

**Multiplication Property of Equality:**

If a=b and c≠0, then ac=bc.
You can multiply or divide both sides of an equation by the same nonzero number and get an equivalent equation.

Example.

④ $$\begin{aligned} 21 &= 3x \\ \frac{21}{3} &= \frac{3x}{3} \\ 7 &= x \end{aligned}$$

**Symmetric Property of Equality:**

If a=b then b=a.

⑤ $$\begin{aligned} 7 &= x \\ x &= 7 \end{aligned}$$

**Substitution Property:**

If a=b, then a may replace b in an equation.
This allows you to check your answer.

⑥ $$\begin{aligned} 28 - x &= 2x + 7 \\ 28 - (7) &= 2(7) + 7 \\ 28 - 7 &= 14 + 7 \\ 21 &= 21 \end{aligned}$$

Thus, the statement is true.

## Steps to Solve Equations:

1. Clear fractions (and decimals) by multiplying every term by the LCD.
2. Simplify each side of the equation by
    a. Removing grouping symbols
    b. Combine like terms
3. Get variable terms together (use the addition property)
4. Get the **variable term** alone (use the addition property)
5. Get the **variable** alone (use the multiplication property)
6. Check your answer (either in a calculator or by pen and pencil using the substitution property)

**Example.**

$$\frac{5}{6}(1-t)-\frac{t}{2}=-\frac{1}{3}(1-3t)$$

1. The denominators in this equation are 6, 2, and 3, so the LCD is 6. Multiply every *term* by 6. Parentheses count as a single term.

$$6\left[\frac{5}{6}(1-t)-\frac{t}{2}\right]=6\left[-\frac{1}{3}(1-3t)\right]$$
$$6\cdot\frac{5}{6}(1-t)-6\cdot\frac{t}{2}=-6\cdot\frac{1}{3}(1-3t)$$
$$5(1-t)-3t=-2(1-3t)$$

2. Simplify by distributing and combining like terms.

$$5(1-t)-3t=-2(1-3t)$$
$$5-5t-3t=-2+6t$$
$$5-8t=-2+6t$$

3. Get the variable together.

$$5-8t=-2+6t$$
$$\underline{\quad +8t \qquad +8t}$$
$$5=-2+14t$$

4. Get the variable term alone, i.e. get the constant terms together on one side.

$$5=-2+14t$$
$$\underline{+2+2}$$
$$7=14t$$

5. Get the variable alone.

$$7=14t$$
$$\overline{14}\ \overline{14}$$
$$\frac{1}{2}=t \quad or \quad t=\frac{1}{2}$$

6. Check.

$$\frac{5}{6}(1-t)-\frac{t}{2}=-\frac{1}{3}(1-3t)$$

$$\frac{5}{6}\left(1-\frac{1}{2}\right)-\frac{1/2}{2}=-\frac{1}{3}\left(1-3\left(\frac{1}{2}\right)\right)$$

$$\frac{5}{6}\left(\frac{1}{2}\right)-\frac{1}{4}=-\frac{1}{3}\left(1-\frac{3}{2}\right)$$

$$\frac{5}{12}-\frac{1}{4}=-\frac{1}{3}\left(-\frac{1}{2}\right)$$

$$\frac{2}{12}=\frac{1}{6}$$

**Problems.**

1. $3x-14x=16-11x-10$
2. $4(k+3)=7k+12-3k$
3. $4(m-6)-m=8(m-3)-5m$
4. $4(x+5)=3(x-4)+x$
5. $6(y-4)=3(y-8)$
6. $\frac{y}{3}-\frac{y}{4}=\frac{1}{6}$
7. $\frac{x+5}{2}+\frac{1}{2}=2x-\frac{x-3}{8}$
8. $\frac{2x+7}{8}-2=x+\frac{x-1}{2}$