1. $3x - 14x = 16 - 11x - 10$ like terms

$-11x = 6 - 11x$ combine variables
$+11x \qquad\qquad +11x$

$$0 = 6 + 0$$ Contradiction

no solution, $\varnothing$

2. $4(k+3) = 7k + 12 - 3k$ distribute, combine like terms

$4k + 12 = 4k + 12$ identity

Solution: x is all real #'s

3. $4(m-6) - m = 8(m-3) - 5m$ distribute

$4m - 24 - m = 8m - 24 - 5m$ combine like terms

$3m - 24 = 3m - 24$ identity

solution: x is all real #'s

4. $4(x+5) = 3(x-4) + x$ distribute

$4x + 20 = 3x - 12 + x$ combine like terms

$4x + 20 = 4x - 12$ combine variables
$-4x \qquad\qquad -4x$

$$0 + 20 = 0 - 12$$ Contradiction

no solution, $\varnothing$

5. $6(y-4) = 3(y-8)$ — distribute

$6y - 24 = 3y - 24$ — addition rule to combine constants.
$\quad +24 \qquad\quad +24$

$\overline{}$

$6y \;=\; 3y$ — combine variables to one side
$-3y \qquad -3y$

$\overline{}$

$\dfrac{3y}{3} = \dfrac{0}{3}$ — divide / multiplication rule

$y = 0 \qquad \{0\}$

6. $\dfrac{y}{3} - \dfrac{y}{4} = \dfrac{1}{6}$

method 1: clear fractions
multiply by LCD = 12

$\dfrac{\overset{4}{\cancel{12}}}{1}\left(\dfrac{y}{\cancel{3}}\right) - \dfrac{\overset{3}{\cancel{12}}}{1}\left(\dfrac{y}{\cancel{4}}\right) = \dfrac{\overset{2}{\cancel{12}}}{1}\cdot\dfrac{1}{\cancel{6}}$

cancel fractions & simplify

$4y - 3y = 2$ — combine like terms

$y = 2 \qquad \{2\}$

$\dfrac{y}{3} - \dfrac{y}{4} = \dfrac{1}{6}$ — method 2: keep fractions

$\dfrac{1}{3}y - \dfrac{1}{4}y = \dfrac{1}{6}$

$\left(\dfrac{1}{3} - \dfrac{1}{4}\right)y = \dfrac{1}{6}$ — combine like terms

$\dfrac{1}{12}y = \dfrac{1}{6}$
$\qquad\qquad\qquad \dfrac{1}{3} - \dfrac{1}{4} = \dfrac{4}{12} - \dfrac{3}{12} = \dfrac{1}{12}$

$\dfrac{\cancel{12}}{1}\cdot\dfrac{1}{\cancel{12}}y = \dfrac{1}{\cancel{6}}\cdot\dfrac{\overset{2}{\cancel{12}}}{1}$ — multiplication rule

$y = 2 \qquad \{2\}$

7. $\dfrac{x+5}{2} + \dfrac{1}{2} = 2x - \dfrac{x-3}{8}$ method 1: clear fractions

LCD = 8

$\dfrac{\cancel{8}^4}{1}\left(\dfrac{x+5}{\cancel{2}}\right) + \dfrac{\cancel{8}^4}{1}\cdot\dfrac{1}{\cancel{2}} = \dfrac{8}{1}\cdot 2x - \dfrac{\cancel{8}}{1}\left(\dfrac{x-3}{\cancel{8}}\right)$ reduce

$4(x+5) + 4 = 16x - (x-3)$ distribute

$4x + 20 + 4 = 16x - x + 3$ combine like terms

$4x + 24 = 15x + 3$ combine variables

$\underline{-15x \qquad\qquad -15x}$

$-11x + 24 = 3$ combine constants

$\underline{\qquad -24 \quad -24}$

$\dfrac{-11x}{-11} = \dfrac{-21}{-11}$ multiplication rule

$x = \dfrac{21}{11}$ $\left\{\dfrac{21}{11}\right\}$ conditional

$\dfrac{x+5}{2} + \dfrac{1}{2} = 2x - \dfrac{x-3}{8}$ Method 2: separate fractions

$\dfrac{x}{2} + \dfrac{5}{2} + \dfrac{1}{2} = 2x - \left(\dfrac{x}{8} - \dfrac{3}{8}\right)$ distribute

$\dfrac{1}{2}x + \dfrac{5}{2} + \dfrac{1}{2} = 2x - \dfrac{1}{8}x + \dfrac{3}{8}$ combine like terms

$\dfrac{1}{2}x + 3 = \dfrac{15}{8}x + \dfrac{3}{8}$ $\dfrac{5}{2} + \dfrac{1}{2} = \dfrac{6}{2} = 3$

$\underline{\quad -3 \qquad\qquad -3}$ $2 - \dfrac{1}{8} = \dfrac{16}{8} - \dfrac{1}{8} = \dfrac{15}{8}$

combine constants

$\dfrac{1}{2}x = \dfrac{15}{8}x - \dfrac{21}{8}$ $\dfrac{3}{8} - 3 = \dfrac{3}{8} - \dfrac{24}{8} = -\dfrac{21}{8}$

$\underline{-\dfrac{15}{8}x \qquad -\dfrac{15}{8}x}$ combine variables

$-\dfrac{11}{8}x = -\dfrac{21}{8}\left.\right]-\dfrac{8}{11}$ $\dfrac{1}{2} - \dfrac{15}{8} = \dfrac{4}{8} - \dfrac{15}{8} = -\dfrac{11}{8}$

multiply by $\left(-\dfrac{8}{11}\right)$

$x = \dfrac{21}{11}$ $\left\{\dfrac{21}{11}\right\}$

8. $\dfrac{2x+7}{8} - 2 = x + \dfrac{x-1}{2}$ Method 1: clear fractions   ④

$\dfrac{8}{1} \cdot \left(\dfrac{2x+7}{8}\right) - 8 \cdot 2 = 8x + \dfrac{8^{4}}{1}\left(\dfrac{x-1}{2}\right)$ reduce

$2x + 7 - 16 = 8x + 4(x-1)$ distribute

$2x + 7 - 16 = 8x + 4x - 4$ Combine like terms

$\begin{aligned} 2x - 9 &= 12x - 4 \\ +9 & \qquad\quad +9 \end{aligned}$ combine constants

    combine variables

$\begin{aligned} 2x &= 12x + 5 \\ -12x & \quad\; -12x \end{aligned}$

$\dfrac{-10x}{-10} = \dfrac{5}{-10}$ multiplication rule

$x = -\dfrac{1}{2}$ $\left\{-\dfrac{1}{2}\right\}$ Conditional

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$\dfrac{2x+7}{8} - 2 = x + \dfrac{x-1}{2}$ Method 2: separate fractions

$\dfrac{2x}{8} + \dfrac{7}{8} - 2 = x + \dfrac{x}{2} - \dfrac{1}{2}$ Combine like terms

$\begin{aligned} \tfrac{1}{4}x - \tfrac{9}{8} &= \tfrac{3}{2}x - \tfrac{1}{2} \\ + \tfrac{9}{8} & \qquad\quad +\tfrac{9}{8} \end{aligned}$ $\dfrac{7}{8} - 2 = \dfrac{7}{8} - \dfrac{16}{8} = \dfrac{-9}{8}$

    $1 + \dfrac{1}{2} = \dfrac{2}{2} + \dfrac{1}{2} = \dfrac{3}{2}$

    Combine constants

$\begin{aligned} \tfrac{1}{4}x &= \tfrac{3}{2}x + \tfrac{5}{8} \\ -\tfrac{3}{2}x & \qquad -\tfrac{3}{2}x \end{aligned}$ $-\dfrac{1}{2} + \dfrac{9}{8} = \dfrac{-4}{8} + \dfrac{9}{8} = \dfrac{5}{8}$

    Combine variables

$-\dfrac{4}{5} \cdot -\dfrac{5}{4}x = \dfrac{5}{8} \cdot -\dfrac{4}{5}$ $\dfrac{1}{4} - \dfrac{3}{2} = \dfrac{1}{4} - \dfrac{6}{4} = \dfrac{-5}{4}$

    multiplication rule

$x = -\dfrac{1}{2}$ $\left\{-\dfrac{1}{2}\right\}$ Conditional