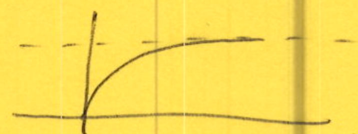
192 Homework #9 Key

1a. $a_n = \dfrac{3n}{n+2}$ $\quad f(x) = \dfrac{3x}{x+2}$ $\quad f'(x) = \dfrac{3(x+2) - 3x(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

bounded above by 3 (and below by 0) $f' > 0$ for all x; always increasing.

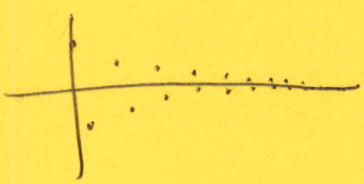
yes, its monotonic



converges to $\lim\limits_{n\to\infty} \dfrac{3n}{n+2} = 3$

b. $c_n = \left(-\dfrac{2}{3}\right)^n$ no alternates between positive & negative values

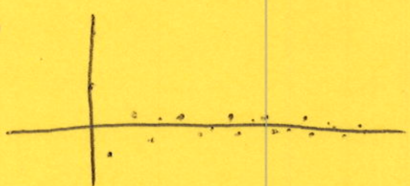
bounded between 1 above and $-2/3$ below. Converges to



$\lim\limits_{n\to\infty} \left(-\dfrac{2}{3}\right)^n = 0$

c. $e_n = \dfrac{\sin n}{n}$ $\quad 0 = \lim\limits_{n\to\infty} \dfrac{-1}{n} \le \lim\limits_{n\to\infty} \dfrac{\sin n}{n} \le \lim\limits_{n\to\infty} \dfrac{1}{n} = 0$

converges



not monotonic bounded between 1 and -1

d. $a_n = \sqrt{\dfrac{n+1}{9n+1}}$ $\quad \lim\limits_{n\to\infty} \sqrt{\dfrac{n+1}{9n+1}} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{3}$ converges

bounded above by 1 below by 1/3

$f(x) = \left(\dfrac{x+1}{9x+1}\right)^{1/2} \Rightarrow f'(x) = \dfrac{1}{2}\left[\dfrac{x+1}{9x+1}\right]^{-1/2} \cdot \dfrac{(9x+1) - 9(x+1)}{(9x+1)^2}$

$= \dfrac{1}{2}\sqrt{\dfrac{9x+1}{x+1}}\left[\dfrac{\cancel{9x}+1-\cancel{9x}-9}{(9x+1)^2}\right] = \dfrac{-4}{(9x+1)^{3/2}(x+1)}$ decreasing for all $x > -1$

Monotonic.

e. $a_n = \arctan(\ln n)$ $\quad \lim\limits_{n\to\infty} \arctan(\ln n) = \pi/2$

$f(x) = \arctan(\ln x) \Rightarrow f'(x) = \dfrac{1}{1+\ln^2 x} \cdot \dfrac{1}{x} > 0$ for all $x \ge 1$

yes, monotonic always increasing

bounded above by $\pi/2$, below by 0

192 Homework #9 Key

1.f. $a_n = n - \sqrt{n+1}\sqrt{n+3}$ $\quad f(x) = x - \sqrt{\cancel{x^2}+4\cancel{x}+3}\,\dfrac{(x+\sqrt{x^2+4x+3})}{x+\sqrt{x^2+4x+3}} =$

$$f(x) = \frac{x^2-(x^2+4x+3)}{x+\sqrt{x^2+4x+3}} = \frac{-4x-3}{x+\sqrt{x^2+4x+3}} \quad \lim_{x\to\infty} \frac{-4x-3}{x+\sqrt{x^2+4x+3}} = \frac{-4}{1+1} = -2$$

converges

bounded by 0 and -2 $\quad f'(x) = 1 - \frac{1}{2}(x^2+4x+3)^{-1/2}(4+2x)$

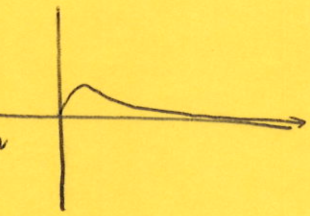
$$= 1 - \frac{x+2}{\sqrt{x^2+4x+3}} = \frac{\sqrt{x^2+4x+3}-(x+2)}{\sqrt{x^2+4x+3}} = 0$$

$\sqrt{x^2+4x+3} = x+2$

$x^2+4x+3 \neq x^2+4x+4$ never zero always same sign

$x=1 \Rightarrow 1 - \dfrac{1+2}{\sqrt{1+4+3}} = 1 - \dfrac{3}{\sqrt{8}} < 0$ always decreasing

1g. $b_n = n e^{-n/2}$ $\quad \lim_{n\to\infty} n e^{-n/2} = 0$ converges



max at $n=2$ monotonically decreasing thereafter

bounded by 1 above and 0 below

$f(x) = x e^{-x/2} \Rightarrow f'(x) = e^{-x/2} - \dfrac{x}{2}e^{-x/2} = e^{-x/2}(1-\tfrac{1}{2}x) \Rightarrow = 0$ when $x=2$

1h. $d_n = \cos\left(\frac{n\pi}{2}\right) =$ does not converge

$= \ldots 0, 1, 0, -1, 0, 1, 0, -1, \ldots$ etc forever

bounded but not monotonic

i. $a_n = \tan\left(\dfrac{2n\pi}{1+8n}\right)$ $\quad f(x) = \tan\left(\dfrac{2\pi x}{1+8x}\right) \Rightarrow f'(x) = \sec^2\left(\dfrac{2\pi x}{1+8x}\right)\cdot\dfrac{2\pi(1+8x)-8(2\pi x)}{(1+8x)^2}$

$= \sec^2\left(\dfrac{2\pi x}{1+8x}\right)\dfrac{2\pi}{(1+8x)^2} > 0$ always increasing monotonic

$\lim_{n\to\infty} \tan\left(\dfrac{2n\pi}{1+8n}\right) = \tan\left(\dfrac{2\pi}{8}\right) = \tan\left(\dfrac{\pi}{4}\right) = 1$ bounded by 0 and 1

j. $a_n = \dfrac{(-1)^{n+1}n}{n+\sqrt{n}} = \dfrac{(-1)^{n+1}}{1+\frac{1}{\sqrt{n}}}$ $\quad \lim_{n\to\infty} \dfrac{(-1)^{n+1}}{1+\frac{1}{\sqrt{n}}} =$ DNE alternates signs

limit of + terms is 1, neg terms is -1

not monotonic, diverges

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1k. $a_n = \sqrt[n]{2^{1+3n}} = \sqrt[n]{2 \cdot 2^{3n}} = \sqrt[n]{2} \cdot 2^3 = 8\sqrt[n]{2}$

$\lim_{n \to \infty} 8\sqrt[n]{2} = 8$ bounded above by 1 and below by 8

$f(x) = 8 \cdot 2^{1/x} \Rightarrow f'(x) = 8 \cdot 2^{1/x} \cdot \left(-\frac{1}{x^2}\right) \cdot \ln 2 < 0$ for all x

always decreasing

2 a. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ telescoping $\frac{A}{n} + \frac{B}{n+2} = An + 2A + Bn = 0n + 1$

$A + B = 0 \Rightarrow 2A = 1 \Rightarrow A = 1/2$

$B = -1/2$

$= \frac{1}{2}\sum_{n=1}^{\infty}\left[\frac{1}{n} - \frac{1}{n+2}\right] = \frac{1}{2}\left[1 + \frac{1}{2} - \lim_{n\to\infty}\left(\frac{1}{n+2} + \frac{1}{n+3}\right)\right] =$

$\frac{1}{2}\left[\frac{3}{2}\right] = \frac{3}{4}$

b. $\sum_{n=1}^{\infty} \sin(n^n)$ nth term test $\lim_{n\to\infty} \sin(n^n)$ does not exist

diverges

c. $\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right) = \sum_{n=1}^{\infty}\left[\ln n+1 - \ln n\right] = -\sum_{n=1}^{\infty}\left[\ln n - \ln(n+1)\right]$ telescoping

$= -\left[\ln 1 - \lim_{n\to\infty} \ln(n+1)\right] = -\infty$ diverges

d. $\sum_{n=1}^{\infty} \frac{1}{n^2+n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2(n+1)}$ $\quad \frac{A}{n} + \frac{B}{n^2} + \frac{C}{n+1} \Rightarrow An^2 + An + Bn + B + Cn^2 = 1$

$A + C = 0$

$A + B = 0$

$B = 1 \Rightarrow A = -1, C = 1$

$= \underbrace{\sum_{n=1}^{\infty}\frac{1}{n^2}}_{\text{converges by integral or p-series}} - \sum_{n=1}^{\infty}\left[\frac{1}{n} - \frac{1}{n+1}\right]$ telescoping

$+ (-1)\left[1 - \lim_{n\to\infty}\frac{1}{n+1}\right] = \left[\sum_{n=1}^{\infty}\frac{1}{n^2}\right] - 1$ converges

e. $\sum_{n=1}^{\infty}\left(\cos\frac{1}{n^2} - \cos\frac{1}{(n+1)^2}\right)$ telescoping

$\cos(1) - \lim_{n\to\infty}\cos\left(\frac{1}{(n+1)^2}\right) = \cos(1) - \cos\left[\lim_{n\to\infty}\frac{1}{(n+1)^2}\right] = \cos(1) - \cos 0 =$

$\cos(1) - 1$

Converges

192 Homework #9 Key

2f. $\sum_{n=2}^{\infty} \frac{1}{n^3-n} = \sum_{n=2}^{\infty} \frac{1}{n(n+1)(n-1)} \Rightarrow \frac{A}{n} + \frac{B}{n+1} + \frac{C}{n-1} \Rightarrow A(n^2-1) + B(n(n-1)) + C(n)(n+1) =$

$An^2 - A + Bn^2 - Bn + Cn^2 + Cn = 1$

$A+B+C=0 \Rightarrow B+C=1$

$-A = 1 \Rightarrow A = -1$

$-B+C=0$

$2C = 1 \Rightarrow C = 1/2$

$B = 1/2$

$= \sum_{n=2}^{\infty} \left[\frac{-1}{n} + \frac{1/2}{n+1} + \frac{1/2}{n-1}\right] = \sum_{n=2}^{\infty} \left(\left[\frac{1/2}{n-1} - \frac{1/2}{n}\right] - \left[\frac{1/2}{n} - \frac{1/2}{n+1}\right]\right)$

telescoping

$= \frac{1}{2}\sum_{n=2}^{\infty}\left[\frac{1}{n-1} - \frac{1}{n}\right] - \frac{1}{2}\sum_{n=2}^{\infty}\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$= \frac{1}{2}\left[1 - \lim_{n\to\infty} \overset{\to 0}{\frac{1}{n}}\right] - \frac{1}{2}\left[\frac{1}{2} - \lim_{n\to\infty} \overset{\to 0}{\frac{1}{n+1}}\right] = \frac{1}{2}[1] - \frac{1}{2}\left[\frac{1}{2}\right] = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

Converges

g. $\sum_{n=0}^{\infty} 5\left(-\frac{1}{3}\right)^{n-1} = \sum_{n=0}^{\infty} 5(-3)(-1/3)^n$ geometric $= \frac{-15}{1-(-1/3)} = \frac{-15}{4/3} =$

$-15 \cdot \frac{3}{4} = -\frac{45}{4}$ Converges $|(-1/3)| < 1$

h. $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$ nth term test $\lim_{n\to\infty} \frac{n+1}{2n-1} = \frac{1}{2} \neq 0$ diverges

i. $\sum_{n=0}^{\infty} \frac{4}{2^n} = \sum_{n=0}^{\infty} 4\left(\frac{1}{2}\right)^n$ geometric $(1/2) < 1$ $\frac{4}{1-1/2} = \frac{4}{1/2} = 8$

j. $\sum_{n=2}^{\infty} \ln\left(\frac{n}{n+1}\right)$ is the negative of 2c. diverges by telescoping

k. $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ $\frac{A}{n-1} + \frac{B}{n+1} \Rightarrow An + A + Bn - B = 2$

$A+B=0$

$A-B=2$

$2A = 2 \Rightarrow A = 1,\ B = -1$

$\sum_{n=2}^{\infty}\left[\frac{1}{n-1} - \frac{1}{n+1}\right]$ telescoping

$\frac{1}{1} + \frac{1}{2} - \lim_{n\to\infty}\left(\frac{1}{n+1} + \overset{\to 0}{\frac{1}{n+2}}\right) = \frac{3}{2}$ Converges

l. $\sum_{n=1}^{\infty}\left(e^{\frac{1}{n}} - e^{\frac{1}{n+1}}\right)$ telescoping $e^{\frac{1}{1}} - \lim_{n\to\infty} e^{\overset{\to 0}{\frac{1}{n+1}}} = e - 1$ converges

192 Homework #9 Key

3.a. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ $\quad \frac{\ln n}{n^2} < \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$ Converges by p-series,

$\therefore$ Converges by direct Comparison or use integral test

b. $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$ $\quad \int_1^{\infty} \frac{n}{n^4+1}\,dn$ $\quad u = n^2$, $du = 2n\,dn$ $\quad \int_1^{\infty} \frac{1/2}{u^2+1}\,du = \frac{1}{2}\arctan n^2 \Big|_1^{\infty}$

$= \frac{\pi}{4} - \frac{\pi}{8}$ Converges by integral test

c. $\sum_{n=1}^{\infty} \left(\frac{\sin n}{n}\right)^2 < \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by direct comparison

d. $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n^{\pi}}} = \sum_{n=1}^{\infty} \frac{2}{n^{\pi/2}}$ $\quad p > 1 \therefore$ Converges by p-series.

e. $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n^3}\right) = \sum_{n=1}^{\infty} \frac{1}{n^2} - \sum_{n=1}^{\infty} \frac{1}{n^3}$ Converges (both) by p-series

f. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ $\quad \int_2^{\infty} \frac{1}{x \ln x}\,dx$ $\quad \ln x = u$, $\frac{1}{x}dx = du$ $\quad \int \frac{1}{u}\,du \Rightarrow \ln(\ln x)\Big|_2^{\infty} = \infty \quad \frac{1}{\ln(n2))}$

diverges by integral test

g. $\sum_{n=3}^{\infty} \frac{1}{n \ln n \ln(\ln n)}$ $\quad \int_3^{\infty} \frac{1}{x \ln x \ln(\ln x)}\,dx$ $\quad u = \ln(\ln x)$, $du = \frac{1}{x \ln x}\,dx$ $\quad \int \frac{1}{u}\,du \Rightarrow$

$\ln(\ln(\ln n))\Big|_3^{\infty} = \infty$ diverges by integral test

h. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series $p \leq 1$

i. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{(\ln n)^2}} = \sum_{n=1}^{\infty} \frac{1}{n \ln^{2/3} n}$ $\quad \int_2^{\infty} \frac{1}{x \ln^{2/3} x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int \frac{1}{u^{2/3}}\,du$

$\Rightarrow \ln^{1/3} x \Big|_2^{\infty} = \infty$ diverges

4a. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ $\quad \int_2^{\infty} \frac{1}{x(\ln x)^p}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int \frac{1}{u^p}\,du <$ converges $p > 1$; diverges $p \leq 1$

192 Homework #9 Key

4b. $\sum_{n=1}^{\infty} \frac{n}{(1+n^2)^p}$ $\quad \int \frac{x}{(1+x^2)^p}dx$ $\quad u = 1+x^2$, $du = 2x\,dx$ $\quad \frac{1}{2}\int \frac{1}{u^p}du$ < converges $p>1$, diverges $p \le 1$

c. $\sum_{n=2}^{\infty} \frac{\ln n}{n^p}$ $\quad \int \frac{\ln x}{x^p}dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad dv = x^{-p}$, $v = \frac{1}{-p+1}x^{1-p}$

$\frac{1}{1-p}x^{1-p}\cdot \ln x - \frac{1}{1-p}\int x^{-p}dx = \frac{1}{1-p}x^{1-p}\ln x - \frac{1}{(1-p)^2}x^{1-p}\Big|_2^{\infty}$ $\quad p>1$ converges, $p \le$ diverges

5a. $\sum_{n=1}^{\infty} \frac{1}{n^4}$ $\quad \int_N^{\infty} \frac{1}{n^4}dn = \frac{n^{-3}}{-3}\Big|_N^{\infty} = -\frac{1}{3}\left[0 - \frac{1}{N^3}\right] \doteq \frac{1}{3N^3} = .001$

$3N^3 \ge 1000$ $\quad N \ge 6.933\ldots$ $\quad$ Error term is 10 $\quad$ stop at $n=9$

b. $\sum_{n=1}^{\infty} \frac{2}{n^2+5}$ $\quad 2\int_N^{\infty} \frac{1}{x^2+5}dx = \underbrace{2\arctan\left(\frac{x}{\sqrt{5}}\right)}_{2(\pi/2) = \pi} - 2\arctan\left(\frac{N}{\sqrt{5}}\right) < .001$

$N = 4473$

c. $\sum_{n=2}^{\infty} e^{-n/2}$ $\quad \int_N^{\infty} e^{-x/2}dx = -2e^{-x/2}\Big|_N^{\infty} = 2e^{-N/2} < .001$

$.0005 > e^{-N/2} \Rightarrow 2000 < e^{N/2}$ $\quad N > 15.2 \Rightarrow N = 16$

6a. $2 + \frac{516}{1000}\sum_{n=0}^{\infty}\left(\frac{1}{1000}\right)^n = 2 + \frac{516/1000}{1-\frac{1}{1000}} = 2 + \frac{516/\cancel{1000}}{999/\cancel{1000}} = 2 + \frac{516}{999} = \frac{838}{333}$

b. $7 + \frac{12345}{100000}\sum_{n=0}^{\infty}\left(\frac{1}{100000}\right)^n = 7 + \frac{12345/100,000}{1-\frac{1}{100,000}} = 7 + \frac{12345/100,000}{99,999/100,000} = 7 + \frac{12345}{99,999}$

$= \frac{712338}{99,999} = \frac{237446}{33,333}$

c. $10 + \frac{1}{10} + \frac{35}{1000}\sum_{n=0}^{\infty}\left(\frac{1}{100}\right)^n = 10 + \frac{1}{10} + \frac{35/1000}{1-\frac{1}{100}} = 10 + \frac{1}{10} + \frac{35/1000}{99/100} =$

$10 + \frac{1}{10} + \frac{35}{\cancel{1000}}\cdot\frac{\cancel{100}}{99} = 10 + \frac{1}{10} + \frac{35}{990} = \frac{5017}{495}$

192 Homework #9 Key

7a. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} - 1$

b. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{n}\right)^4 = 81\sum_{n=1}^{\infty} \frac{1}{n^4} = 81\left(\frac{\pi^4}{90}\right) = \frac{9\pi^4}{10}$

c. $\displaystyle\sum_{n=3}^{\infty} \frac{1}{(n+1)^2} = \sum_{k=4}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} - \underset{k=1}{1} - \underset{k=2}{\frac{1}{4}} - \underset{k=3}{\frac{1}{9}}$

$k = n+1$

d. $\displaystyle\sum_{n=5}^{\infty} \frac{1}{(n-2)^4} = \sum_{k=3}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90} - \underset{k=1}{1} - \underset{k=2}{\frac{1}{16}}$

$n-2 = k$

e. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{4}\left(\frac{\pi^2}{6}\right) = \frac{\pi^2}{24}$

8a. $a_n = \dfrac{1}{(n+1)!}$ $n=0$ $\quad 1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \ldots$

b. $a_n = \cos\left(\frac{n\pi}{2}\right)$ $n=0$ $\quad 1, 0, -1, 0, 1, \ldots$

c. $a_n = \dfrac{(-1)^n n}{n!+1}$ $n=0$ $\quad 0, \frac{-1}{2}, \frac{2}{3}, \frac{-3}{7}, \frac{4}{25}, \ldots$

d. $a_{n+1} = a_n - a_{n-1}$ $\quad 2, 1, -1, -2, -1, 1, \ldots$

9a. $a_n = \left(-\frac{1}{3}\right)^n \Big|_{n=0}$ b. $\dfrac{(-1)^n n^2}{n+1}\Big|_{n=1} = a_n$ c. $a_n = a_{n-1} + n$, $a_0 = 1$

d. $a_n = 5 + 3n \,\Big|_{n=0}$ e. $a_n = 3^{2^n+1}\Big|_{n=0}$ f. $(2n+1)!\,\Big|_{n=0} = a_n$

10a. $5040 = 7!$ b. $0! = 1$ d. $\dfrac{10!}{3!} = 604{,}800$ c. $\binom{12}{3} = {}_{12}C_3 = 220$

e. $\binom{4}{2} = {}_4C_2 = 6$ f. $\dfrac{10!}{4!\,6!} = 210$ g. $\binom{10}{5} = 252 = {}_{10}C_5$

11a. $S_1 = \frac{1}{1} = 1$, $S_2 = 1 + \frac{1}{8} = \frac{9}{8}$ $S_3 = \frac{9}{8} + \frac{1}{27} = \frac{29}{54}$, $S_4 = \frac{29}{54} + \frac{1}{64} = \frac{955}{1728}$

$S_5 = \frac{955}{1728} + \frac{1}{125} \approx .56066\ldots$ $S_6 = .56066 + \frac{1}{216} \approx .56529\ldots$ $S_7 = .56529\ldots + \frac{1}{343} = .568207$

$S_8 = .568207 + \frac{1}{512} = .57016\ldots$ yes. it does appear to converge (p-series)

11b. $S_1 = \frac{1}{\sqrt{5}}$ $S_2 = \frac{1}{\sqrt{5}} + \frac{2}{\sqrt{8}} = 1.1543..$ $S_3 = S_2 + \frac{3}{\sqrt{13}} = 1.986...$

$S_4 = S_3 + \frac{4}{\sqrt{20}} = 2.880797...$ $S_5 = S_4 + \frac{5}{\sqrt{29}} = 3.80927...$, $S_6 = S_5 + \frac{6}{\sqrt{40}} = 4.757...$

$S_7 = S_6 + \frac{7}{\sqrt{53}} = 5.71948...$ $S_8 = S_7 + \frac{8}{\sqrt{68}} = 6.6896...$ no, diverges

fails nth term test

c. $\sum_{n=2}^{\infty} \frac{1}{n(n+2)}$ $S_2 = \frac{1}{8}$, $S_3 = \frac{1}{8} + \frac{1}{15} = \frac{23}{120}$, $S_4 = \frac{23}{120} + \frac{1}{24} = \frac{7}{30}$, $S_5 = \frac{7}{30} + \frac{1}{35}$

$= \frac{11}{42}$, $S_6 = \frac{11}{42} + \frac{1}{48} = \frac{95}{336}$, $S_7 = \frac{95}{336} + \frac{1}{63} = \frac{43}{144}$, $S_8 = \frac{43}{144} + \frac{1}{80} = \frac{14}{45}$

yes, it appears to converge (p-series/direct comparison)
or telescoping

d. $\sum_{n=1}^{\infty} \frac{1}{\ln n + 1}$ $S_1 = 1$, $S_2 = 1 + \frac{1}{\ln 2 + 1} = 1.59...$ $S_3 = S_2 + \frac{1}{\ln 3 + 1} = 2.067...$

$S_4 = S_3 + \frac{1}{\ln 4 + 1} = 2.486...$ $S_5 = S_4 + \frac{1}{\ln 5 + 1} = 2.8694...$, $S_6 = S_5 + \frac{1}{\ln 6 + 1} = 3.227...$

$S_7 = S_6 + \frac{1}{\ln 7 + 1} = 3.56705...$, $S_8 = S_7 + \frac{1}{\ln 8 + 1} = 3.89179...$ diverges

by direct comparison w/ harmonic series

e. $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ telescoping, converges $S_1 = \left(\frac{1}{1} - \frac{1}{\sqrt{2}}\right)$, $S_2 = 1 - \frac{1}{\sqrt{3}}$

$S_3 = 1 - \frac{1}{\sqrt{4}} = \frac{1}{2}$, $S_4 = 1 - \frac{1}{\sqrt{5}}$, $S_5 = 1 - \frac{1}{\sqrt{6}}$, $S_6 = 1 - \frac{1}{\sqrt{7}}$, $S_7 = 1 - \frac{1}{\sqrt{8}}$

$S_8 = 1 - \frac{1}{\sqrt{9}} = 1 - \frac{1}{3} = \frac{2}{3}$.

f. $\sum_{n=1}^{\infty} \sqrt[n]{2}$ diverges by nth term test $S_1 = 2$, $S_2 = 2 + \sqrt{2}$, $S_3 = S_2 + \sqrt[3]{2}$

≈ 4.674, $S_4 = S_3 + \sqrt[4]{2} \approx 5.86$, $S_5 = S_4 + \sqrt[5]{2} \approx 7.012...$, $S_6 = S_5 + \sqrt[6]{2} \approx 8.13...$

$S_7 = S_6 + \sqrt[7]{2} \approx 9.23859...$, $S_8 = S_7 + \sqrt[8]{2} \approx 10.329...$

192 Homework #9 key

12a. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} = -\frac{1}{3}\sum_{n=1}^{\infty} \left(\frac{-3}{4}\right)^n = \frac{-1/3}{1-(-3/4)} - 1 = \frac{-1/3}{7/4} - 1 = -\frac{1}{3}\cdot\frac{4}{7} - 1 = \frac{-4}{21} - 1$

$\uparrow$ n=0 term missing $= \frac{-25}{21}$

Converges $|(-3/4)| < 1$

b. $\sum_{n=0}^{\infty} \frac{e^n}{3^{n-1}} = \frac{1}{3^{-1}}\sum_{n=0}^{\infty} \frac{e^n}{3^n} = 3\sum_{n=0}^{\infty}\left(\frac{e}{3}\right)^n$ converges $\frac{e}{3} < 1$

$\frac{3}{1-\left(\frac{e}{3}\right)} \approx 31.946821\ldots$

c. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \ldots \quad 3\left(1 - \frac{4}{3} + \frac{16}{9} - \frac{64}{27} + \ldots\right) = 3\sum_{n=0}^{\infty}\left(-\frac{4}{3}\right)^n$

diverges since $|-4/3| > 1$

d. $\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^n \quad \frac{1}{\sqrt{2}} < 1$ converges $\frac{1}{1-\frac{1}{\sqrt{2}}} \approx 3.4142\ldots$

e. $\sum_{k=1}^{\infty}(\cos 1)^k \quad \cos 1 \approx .54 < 1$ converges $\frac{1}{1-\cos 1} \approx 2.1753$

f. $\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \cdots$ (treat this like sum of 2 series)

$\left(\frac{1}{3} + \frac{1}{27} + \frac{1}{243} + \ldots\right) + \left(\frac{2}{9} + \frac{2}{81} + \frac{2}{729} + \ldots\right)$

$\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n + \frac{2}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n$ converges $\frac{1}{9} < 1 \quad \frac{1/3}{1-1/9} + \frac{2/9}{1-1/9} = \frac{1}{3}\cdot\frac{9}{8} + \frac{2}{9}\cdot\frac{9}{8}$

$= \frac{3}{8} + \frac{1}{4} = \frac{5}{8}$

or $\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \frac{1}{729} + \ldots\right) + \left(\frac{1}{9} + \frac{1}{81} + \frac{1}{729} + \ldots\right) =$

$\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n + \frac{1}{9}\sum_{n=0}^{\infty}\left(\frac{1}{9}\right)^n = \frac{1/3}{1-1/3} + \frac{1/9}{1-1/9} = \frac{1}{3}\cdot\frac{3}{2} + \frac{1}{9}\cdot\frac{9}{8} = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

13a. $\sum_{n=0}^{\infty} 4\left(\frac{x-3}{4}\right)^n$ converges when $\left|\frac{x-3}{4}\right| < 1 \quad -1 < \frac{x-3}{4} < 1 \Rightarrow$

$-4 < x - 3 < 4 \Rightarrow -1 < x < 7 \quad (-1, 7)$

$+3 \quad +3 \quad +3$

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13b. $\sum_{n=0}^{\infty} \frac{2^n}{x^n}$ $\quad \left|\frac{2}{x}\right|<1 \quad -1<\frac{2}{x}<1 \quad -1<\frac{2}{x} \Rightarrow -1>\frac{x}{2} \Rightarrow -2>x$

$\frac{2}{x}<1 \Rightarrow \frac{x}{2}>1 \Rightarrow x>2$

all values less than -2 and greater than 2

c. $\sum_{n=0}^{\infty} e^{nx} = \sum_{n=0}^{\infty} (e^x)^n \quad e^x<1 \Rightarrow x<0$

d. $\sum_{n=0}^{\infty} (-5)^n x^n = \sum_{n=1}^{\infty} (-5x)^n \Rightarrow |-5x|<1 \quad -1<5x<1$

$(-1/5, 1/5)$

e. $\sum_{n=0}^{\infty} \left(\frac{\sin x}{3}\right)^n \quad \left|\frac{\sin x}{3}\right|<1 \Rightarrow$ for all x since $|\sin x|<1$

for all x