MAT 192, Quiz #6, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Solve the differential equation $\frac{dy}{dt} = \frac{e^y \sin^2 t}{y \sec t}$.

$$\int e^{-y} y \, dy = \int \cos t \sin^2 t \, dt$$

$u = y \quad dv = e^{-y} dy$

$du = dy \quad v = -e^{-y}$

$$-ye^{-y} + \int e^{-y} dy = \frac{1}{3}\sin^3 t + C$$

$$-ye^{-y} - e^{-y} = \frac{1}{3}\sin^3 t + C$$

2. Suppose a population satisfies $\frac{dP}{dt} = 0.4P - 0.001P^2, P(0) = 50$. Find $P(t)$.

$$\frac{-1}{1000}(P^2 - 400P) = -\frac{P}{1000}(P-400)$$

$$\frac{dP}{P(P-400)} = -\frac{1}{1000} dt$$

$$\frac{A}{P} + \frac{B}{P-400}$$

$$AP - 400A + BP = 1$$

$$A + B = 0$$

$$-400A = 1 \Rightarrow A = \frac{-1}{400} \qquad B = \frac{1}{400}$$

$$\frac{-1}{400}\int \frac{dP}{P} + \frac{1}{400}\int \frac{dP}{P-400} = \int -\frac{1}{1000} dt$$

$$-\frac{1}{400}\ln P + \frac{1}{400}\ln|P-400| = -\frac{1}{1000}t + C$$

$$\ln\left(\frac{P-400}{P}\right)^{1/400} = -\frac{1}{1000}t + C$$

$$\frac{P-400}{P} = Ae^{\frac{-1}{1000}t} \Rightarrow P - 400 = P \cdot Ae^{-1/1000 t}$$

$$P - PAe^{-1/1000 t} = 400$$

$$P(1 - Ae^{-1/1000 t}) = 400$$

$$P(t) = \frac{400}{1 - Ae^{-1/1000 t}}$$

$$50 = \frac{400}{1-A}$$

$$50 - 50A = 400$$

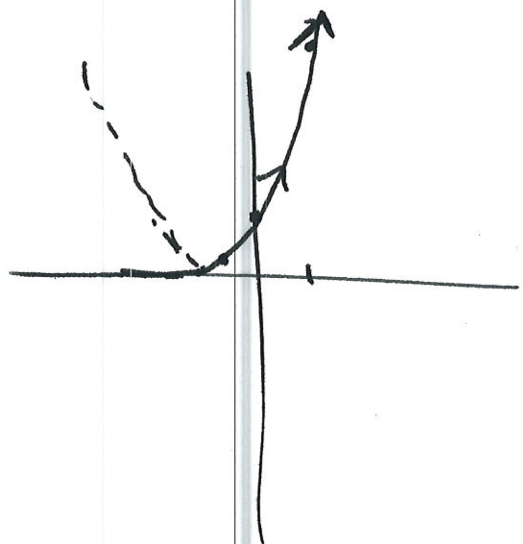
$$\frac{-50A}{-50} = \frac{350}{-50}$$

$$A = -7$$

$$\boxed{P(t) = \frac{400}{1 + 7e^{-1/1000 t}}}$$

3. Sketch the parametric set of equations $x = e^t - 1, y = e^{2t}$. Indicate an arrow on the curve for the direction of increasing t, then rewrite the equation in nonparametric/Cartesian form.

t	x	y
0	0	1
1	$e-1$	e^2
-1	$\frac{1}{e}-1$	e^{-2}



$x+1 = e^t$

$\ln|x+1| = t$

$y = e^{2\ln|x+1|}$

$y = e^{\ln(x+1)^2}$

$y = (x+1)^2$ →

Parametric is right half of graph