MAT 192, Quiz #8, Fall 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find the length of arc of the function $y = \ln(\sec x)$ on $\left[0, \frac{\pi}{4}\right]$.

$$y' = \frac{1}{\cancel{\sec x}} \cdot \cancel{\sec x} \tan x = \tan x$$

$$\int_0^{\pi/4} \underbrace{\sqrt{1+\tan^2 x}}_{\sqrt{\sec^2 x}}\, dx = \int_0^{\pi/4} \sec x\, dx = \ln|\sec x + \tan x|\Big|_0^{\pi/4}$$

$$= \ln|\sqrt{2}+1| - \cancel{\ln|1+0|}^{0} = \ln(\sqrt{2}+1)$$

2. Find the length of arc of the parametric curve defined by $x = e^t \cos t$, $y = e^t \sin t$ on $[0, \pi]$.

$$x' = e^t \cos t + -e^t \sin t$$
$$y' = e^t \sin t + e^t \cos t$$

$$\sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2}$$

$$e^t\sqrt{\cos^2 t - 2\cancel{\cos t \sin t} + \sin^2 t + \sin^2 t + 2\cancel{\sin t \cos t} + \cos^2 t}$$

$$= \sqrt{2}e^t$$

$$\int_0^{\pi} \sqrt{2}e^t\, dt = \sqrt{2}e^t\Big|_0^{\pi} = \boxed{\sqrt{2}\left[e^{\pi}-1\right]}$$

3. Write the first 5 terms of the sequence $a_n = \frac{\ln^2 n}{n}$. Does the sequence appear to converge?

$n=1$ $\frac{\ln(1)^2}{1} = 0$

$n=2$ $\frac{(\ln 2)^2}{2}$

$n=3$

$n=4$

$n=5$

$$\left\{0, \frac{(\ln 2)^2}{2}, \frac{(\ln 3)^2}{3}, \frac{(\ln 4)^2}{4}, \frac{(\ln 5)^2}{5}, \ldots\right\}$$

no, it appears to diverge for these terms, a graph, however  does eventually