MAT 192, Quiz #9, Fall 2016 Name KEY

**Instructions**: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Determine if the series $\sum_{n=1}^{\infty}\left(\cos\left(\frac{1}{n^2}\right)-\cos\left(\frac{1}{(n+1)^2}\right)\right)$. If it converges, find the sum.

$\cos(1) - \cos\frac{1}{4} + \cos\frac{1}{4} - \cos\frac{1}{9} + \cos\frac{1}{9} - \cos\frac{1}{16} + \cdots$ etc

Series is telescoping

$$\sum_{n=1}^{\infty}\left(\cos\left(\tfrac{1}{n^2}\right)-\cos\left(\tfrac{1}{(n+1)^2}\right)\right) = \cos(1) - \lim_{n\to\infty}\cos\left(\tfrac{1}{(n+1)^2}\right) = \boxed{\cos(1)-1}$$

2. Use the integral test to determine if $\sum_{n=0}^{\infty}\frac{1}{n^2+4}$ converges or diverges. If it converges estimate the error after 10 terms.

$$\int_0^{\infty}\frac{1}{n^2+4}\,dn = \int_0^{\infty}\frac{1}{x^2+4}\,dx = \frac{1}{2}\arctan\frac{x}{2}\Big|_0^{\infty}$$

$\frac{1}{2}\left[\frac{\pi}{2}-0\right] = \frac{\pi}{4}$ finite so series converges

$$\int_{10}^{\infty}\frac{1}{x^2+4}\,dx = \frac{1}{2}\arctan\frac{x}{2}\Big|_{10}^{\infty} = \frac{1}{2}\left[\frac{\pi}{2}-\arctan\left[\frac{10}{2}\right]\right] \approx .09869\ldots < .1$$

3. Us the ratio or root test to determine if the series converges.

a. $\sum_{n=1}^{\infty}\left(\frac{n^2+1}{2n^2+1}\right)^n$

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{n^2+1}{2n^2+1}\right)^n} = \lim_{n\to\infty}\frac{n^2+1}{2n^2+1}$$

$= \frac{1}{2} < 1$ converges

root test

b. $\sum_{n=1}^{\infty}\frac{n!}{n^n}$

$$\lim_{n\to\infty}\frac{(n+1)!}{(n+1)^{n+1}}\cdot\frac{n^n}{n!} =$$

$$\lim_{n\to\infty}\frac{\cancel{n!}\,\cancel{(n+1)}\,n^n}{(n+1)^n\,\cancel{(n+1)}\,\cancel{n!}} = \lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n =$$

$$\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)^n = e^{-1} < 1$$

converges