MAT 192, Series Activity 4, Fall 2016

KEY

Group Members ______________________________

Instructions: In groups of 2-4 students, discuss each of the problems below. Discuss solution strategies, appropriate notation, etc. Then each student should write up solutions. Justify all steps. Your group may submit one copy of the assignment with the name of all group members for grading.

1. Determine the convergence or divergence of the series. State which tests you ***could*** use. Explain which test you think is best, and then test the series. If your group can't agree on which test will work best, different group members can apply different tests and then compare your results at the end. If you do that, explain which tests were tried and which turned out to work better.

a. $\sum_{n=1}^{\infty} \frac{1}{n+3^n}$

ratio test could work but math is pretty messy.
limit comparison or direct comparison better w/ $\frac{1}{3^n}$

$\frac{1}{n+3^n} \le \frac{1}{3^n}$

converges by direct comparison

l. $\sum_{n=1}^{\infty} \frac{n^2 2^{n-1}}{(-5)^n} = \sum_{n=1}^{\infty} \frac{n^2 2^n}{2 \cdot (-5)^n}$

ratio or root test are best here

$\lim_{n\to\infty} \sqrt[n]{\left|\frac{n^2 2^n}{2(-5)^n}\right|} = \frac{2}{5} < 1$ converges

could use integral test but needs by parts

b. $\sum_{n=1}^{\infty} \frac{(2n+1)^n}{n^{2n}}$

root test is best
ratio could work w/ more algebra

$\lim_{n\to\infty} \sqrt[n]{\frac{(2n+1)^n}{n^{2n}}} =$

$\lim_{n\to\infty} \frac{2n+1}{n^2} = 0 < 1$ converges

m. $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$

ratio test is best

$\lim_{k\to\infty} \frac{2^{k+1}(k+1)\cancel{k!}}{(k+3)\cancel{(k+2)!}} \cdot \frac{\cancel{(k+2)!}}{2^k \cancel{k!}} = \lim_{k\to\infty} \frac{2(k+1)}{k+3} = 2$

diverges

c. $\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{1}{3^n}\right) = \sum_{n=1}^{\infty} \frac{1}{n^3} + \sum_{n=1}^{\infty} \frac{1}{3^n}$

both converge by p-series & geometric series

can also use integral test (ratio test, root test on 2nd half)

n. $\sum_{n=1}^{\infty} \frac{\sin 2n}{1+2^n}$

direct comparison works best since sin 2n does not converge but is bounded

$\frac{\sin 2n}{1+2^n} < \frac{1}{1+2^n} < \frac{1}{2^n}$

converges by geometric series

d. $\sum_{k=1}^{\infty} \frac{2^{k-1}3^{k+1}}{k^k} = \sum_{k=1}^{\infty} \frac{2^k \cdot 3^k \cdot 3}{2 \cdot k^k} =$

$\frac{3}{2}\sum_{k=1}^{\infty} \frac{6^k}{k^k}$ ratio or root test work well

$\lim_{k\to\infty} \sqrt[k]{\frac{6^k}{k^k}} = \lim_{k\to\infty} \frac{6}{k} = 0 < 1$

Converges

o. $\sum_{n=1}^{\infty} \tan\left(\frac{1}{n}\right)$ limit comparison works best w/ $\frac{1}{n}$

$\lim_{n\to\infty} \frac{\tan(\frac{1}{n})}{\frac{1}{n}} \Rightarrow$ let $\frac{1}{n} = x \Rightarrow$

$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sec^2 x}{1} = 1$

diverges since $\frac{1}{n}$ diverges

e. $\sum_{n=0}^{\infty} \frac{n!}{e^{n^2}}$ ratio test is best

$\lim_{n\to\infty} \frac{(n+1)!}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n!} = \lim_{n\to\infty} \frac{(n+1)}{e^{2n+1}} = 0$

e^{n^2+2n+1}

Converges

p. $\sum_{k=1}^{\infty} \frac{k \ln k}{(k+1)^3}$ $\ln k < \sqrt{k}$
direct comparison w/ $\frac{k\sqrt{k}}{(k+1)^3} < \frac{1}{k^{3/2}}$ converges by p-series

ratio won't work
Could try integrating

f. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh n}$ alternating
integral (for abs. conv. only)
ratio using def. of cosh.

$\lim_{n\to\infty} \frac{1}{\cosh n} = 0$

Converges

q. $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ root test works best

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = L$

$\lim_{n\to\infty} \ln\left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} n \ln\left(\frac{n}{n+1}\right) = \lim_{n\to\infty} \frac{\ln\left(1-\frac{1}{n+1}\right)}{\frac{1}{n}}$

$= \lim_{n\to\infty} \frac{\frac{1}{1-\frac{1}{n+1}} \cdot \frac{1}{(n+1)^2}}{-\frac{1}{n^2}} = -1 = \ln L \quad L = e^{-1} = \frac{1}{e} < 1$

Converges

g. $\sum_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)^n$ root test

$\lim_{n\to\infty} \sqrt[n]{\left(\sqrt[n]{2}-1\right)^n} \lim_{n\to\infty} \sqrt[n]{2} - 1 = 0 < 1$

Converges

r. $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$ root test

$\lim_{n\to\infty} \sqrt[n]{\frac{(n!)^n}{n^{4n}}} = \lim_{n\to\infty} \frac{n!}{n^4} = \infty$

diverges

h. $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$ $< \sum \frac{1}{n^{1.1}}$ comparison

follows since $n^{1.1} < \ln(n)^{\ln n}$

for all values > 20

split $\sum_{n=2}^{20} \frac{1}{(\ln n)^{\ln n}} + \sum_{n=21}^{\infty} \frac{1}{(\ln n)^{\ln n}} < \sum_{n=21}^{\infty} \frac{1}{n^{1.1}}$

finite terms = finite limit

converges by p-series/direct comparison

s. $\sum_{n=1}^{\infty} \frac{(-1)^n e^{n^2}}{n^{n!}}$ ratio or root test

$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n e^{n^2}}{n^{n!}}\right|} = \lim_{n\to\infty} \frac{e^n}{n^{(n-1)!}} = 0$

Converges

i. $\sum_{n=1}^{\infty} \frac{(2n+1)!}{e^{n^2}}$ ratio test

$\lim_{n\to\infty} \frac{(2n+3)!}{e^{n^2+2n+1}} \cdot \frac{e^{n^2}}{(2n+1)!} =$

$\lim_{n\to\infty} \frac{(2n+2)(2n+3)}{e^{2n+1}} = 0$ Converges <1

t. $\sum_{n=2}^{\infty} \frac{\ln^n n}{n!}$ ratio test

$\lim_{n\to\infty} \frac{\ln^{n+1}(n+1)}{(n+1)!} \cdot \frac{n!}{\ln^n n} = \lim_{n\to\infty} \frac{1}{n+1} \cdot \left(\frac{\ln(n+1)}{\ln n}\right)^n \cdot \ln(n+1)$

$\lim_{n\to\infty} \frac{\ln(n+1)}{n+1} \cdot \lim_{n\to\infty} \left(\frac{\ln(n+1)}{\ln n}\right)^n =$

$\lim_{n\to\infty} \frac{\frac{1}{n+1}}{1} = 0 \cdot e^{\lim_{n\to\infty} \frac{n \ln(n+1)}{\ln n}} = 0 \cdot e^n = \infty$

$\frac{1}{n+1}$

diverges

j. $\sum_{n=2}^{\infty} \frac{\ln n!}{e^n}$ ratio test

$\lim_{n\to\infty} \frac{\ln(n+1)!}{e^{n+1}} \cdot \frac{e^n}{\ln(n!)} = \lim_{n\to\infty} \frac{1}{e}\left(\frac{\ln 1+\ln 2+\ln 3+\cdots \ln n+\ln(n+1)}{\ln 1+\ln 2+\ln 3+\cdots \ln n}\right)$

$= \frac{1}{e} \lim_{n\to\infty} \left(1 + \frac{\ln(n+1)}{\ln(n!)}\right) = \frac{1}{e}(1) < 1$

Converges

u. $\sum_{n=0}^{\infty} \frac{(n!)^2}{(3n)!}$ ratio test

$\lim_{n\to\infty} \frac{(n+1)!\,(n+1)!}{(3n+3)!} \cdot \frac{(3n)!}{n!\,n!} =$

$\lim_{n\to\infty} \frac{(n+1)^2}{(3n+1)(3n+2)(3n+3)} = 0 < 1$

Converges

k. $\sum_{n=1}^{\infty} \frac{(-1)^n n!}{1\cdot 3\cdot 5\cdot \ldots\cdot(3n-1)}$ ratio

$\lim_{n\to\infty} \left|\frac{(n+1)!}{1\cdot 3\cdot 5\cdots(3n+1)(3n+4)} \cdot \frac{1\cdot 3\cdot 5\cdots(3n-1)}{n!}\right| =$

$\lim_{n\to\infty} \frac{n+1}{3n+4} = \frac{1}{3} < 1$ Converges

v. $\sum_{n=1}^{\infty} \left(-\frac{3n}{2n+1}\right)^{3n}$ root test

$\lim_{n\to\infty} \sqrt[n]{\left|\left(-\frac{3n}{2n+1}\right)^{3n}\right|} = \lim_{n\to\infty} \left|\left(-\frac{3}{2}\right)^3\right| = \left|-\frac{27}{8}\right|$

$= \frac{27}{8} > 1$

diverges