

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Find a parametric/vector-valued function for the line through $(2, -1, 4)$ and $(4, 6, 1)$.

$$\langle 2, 7, -3 \rangle$$

$$\vec{r}(t) = (2t+2)\hat{i} + (7t-1)\hat{j} + (4-3t)\hat{k}$$

2. Find a plane containing the points $(2, -1, 4)$, $(4, 6, 1)$, $(2, 4, 6)$.

$$\langle 2, 7, -3 \rangle \quad \langle 0, 5, 2 \rangle$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -3 \\ 0 & 5 & 2 \end{vmatrix} = (14+15)\hat{i} - (4-0)\hat{j} + (10-0)\hat{k} \\ \langle 29, -4, 10 \rangle$$

$$29(x-2) - 4(y+1) + 10(z-4) = 0$$

3. Describe the surface modeled by equation $x^2 = y^2 + 4z^2$. Convert the equation to cylindrical and spherical coordinates.

elliptical cone wrapped around the x-axis

$$r^2 \cos^2 \theta = r^2 \sin^2 \theta + 4z^2$$

$$r^2 (\cos^2 \theta - \sin^2 \theta) = 4z^2 \quad \text{cylindrical}$$

$$r^2 \cos 2\theta = 4z^2$$

$$\rho^2 \cos^2 \theta \sin^2 \varphi = \rho^2 \sin^2 \theta \sin^2 \varphi + 4\rho^2 \cos^2 \varphi$$

$$(\cos^2 \theta - \sin^2 \theta) \sin^2 \varphi = 4 \cos^2 \varphi$$

$$\cos 2\theta \tan^2 \varphi = 4 \quad \text{spherical}$$