MAT 202, Quiz #13, Fall 2016 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Solve the linear system of differential equations given by $\vec{x}' = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}\vec{x}$. Write the solution in the form $\vec{x} = c_1\vec{v}_1e^{\lambda_1 t} + c_2\vec{v}_2e^{\lambda_2 t}$ (or using sines and cosines if the eigenvalues are complex). Draw several sample trajectories.

$(1-\lambda)(4-\lambda)+2 = \lambda^2 - 5\lambda + 6 = 0$

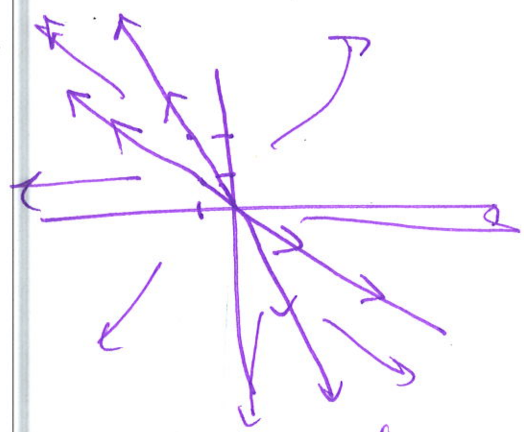
$(\lambda-2)(\lambda-3)=0 \quad \lambda = 2,3$

$\lambda_1 = 2 \quad \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix} \quad \begin{matrix} x_1 = -x_2 \\ x_2 = x_2 \end{matrix} \quad \vec{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\lambda_2 = 3 \quad \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix} \quad \frac{2x_1}{2} = \frac{-x_2}{2}$

$x_1 = -\frac{1}{2}x_2 \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$x_2 = x_2$



Origin repels

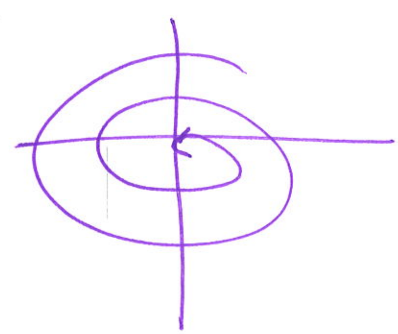
$\vec{x} = c_1\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{2t} + c_2\begin{bmatrix} -1 \\ 2 \end{bmatrix}e^{3t}$

2. Solve the discrete dynamical system $\vec{x}_{n+1} = \begin{bmatrix} .4 & -.25 \\ 1.25 & .5 \end{bmatrix}\vec{x}_n$. Write the solution in the form $\vec{x}_n = c_1\vec{v}_1\lambda_1^n + c_2\vec{v}_2\lambda_2^n$ if the solution is real. Draw several sample trajectories.

$(.4-\lambda)(.5-\lambda) + .3125 = 0$

$\lambda^2 - .9\lambda + .5125 = 0$

$\lambda = \frac{.9 \pm \sqrt{.81 - 2.05}}{2} \approx \frac{.9 \pm 1.11i}{2} = .45 \pm .557i$



$\sqrt{(.45)^2 + (.557)^2} = \sqrt{.5125} = .71589 < 1$

Spirals inward since $\|\lambda\| < 1$