MAT 202, Quiz #6, Fall 2016 Name KEY

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Determine if the following sets are subspaces. If the set is a subspace, prove it. If it is not, find a counterexample.

   a. $H = \left\{ \begin{bmatrix} 0 & a & b \\ c & 0 & c \\ d & f & 1 \end{bmatrix}, a+b+c=d \right\}$

this is not a subspace (all rules fail)
in particular if $a=b=c=d=f=0$ we get $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
So the $\vec{0}$ does not exist in the set.

   b. The set of all $n \times n$ matrices that commute with A, i.e. all B such that $AB = BA$.

i) The 0 matrix is in the set since $0A = A0 = 0$
ii) if we have B, C in set and $AB=BA$ and $CA = AC$,
$(B+C)A = BA+CA = AB+AC = A(B+C)$, so the sum also commutes.
iii) and kB commutes since $kBA = kAB = A(kB)$.

This is a subspace

2. Determine which sets of vectors are independent by inspection and which are not. Explain your reasoning. If the set is dependent, find a minimum spanning set.

   a. $\left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ -5 \end{bmatrix} \right\}$ independent. 2 vectors, not multiples

   b. $\left\{ \begin{bmatrix} 7 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ dependent, $\vec{0}$ vector in set
minimum spanning set is $\left\{ \begin{bmatrix} 7 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ -4 \\ 1 \end{bmatrix} \right\}$

   c. $\left\{ \begin{bmatrix} 3 \\ 4 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 9 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \end{bmatrix} \right\}$ dependent; 5 vectors in $\mathbb{R}^4$

rref $\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -209/83 \\ 0 & 1 & 0 & 0 & 553/83 \\ 0 & 0 & 1 & 0 & -14/83 \\ 0 & 0 & 0 & 1 & 81/83 \end{bmatrix}$

minimum spanning set $\left\{ \begin{bmatrix} 3 \\ 4 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 9 \\ 2 \\ 1 \\ 0 \end{bmatrix} \right\}$